PREFACE

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HONG DUC UNIVERSITY
JOURNAL OF SCIENCE
E5 - Volume 10. 2019

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OZONATION AND CULTIVATION OF GREEN ALGAE *CHLORELLA ZOFINGIENSIS* IN DOMESTIC WASTEWATER

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Received: 13 November 2018/ Accepted: 11 June 2019/ Published: June 2019

Abstract: *Ozone has been used in treatment of municipal wastewater effluents primary for disinfection. However, its powerful capacities to react with organic matter in wastewater led to another application of ozone in water treatment. Heterotrophic cultivation of green algae Chlorella zofingiensis in ozonated waste water was used to remove nutrients and also to collect Astaxanthin. Results of this study shows ozonation did not remove but produce transformation of organic matters; increase capacity of dissolved organic carbon and dissolved nitrogen removal of Chlorella zofingiensis as well. The ozonation - heterotrophic cultivation of Chlorella zofingiensis process could be used in domestic wastewater treatment. However, as the dual-goal of algae cultivation, astaxanthin production in this process was not effective.*

Keywords: *Ozonation, algae cultivation, domestic wastewater, astaxanthin*.

1. Introduction

Domestic (also called sanitary) wastewater is wastewater discharged from residences and from commercial, institutional, and similar facilities. It is handed by wastewater treatment plans and discharged into received water bodies (rivers, sea, etc). General terms used to describe different degrees of treatment are preliminary, primary, secondary, and tertiary and/or advanced wastewater treatment. In some countries, disinfection to remove pathogens sometimes follows the last treatment step (FAO). Disinfection is used in water treatment process to reduce pathogens to acceptable level. There are three normal categories of human enteric pathogens: bacteria, viruses, and amebic cysts. Powerful disinfectant must destroy all three. The common disinfection process using in wastewater treatment are chlorination, ozonation, and ultraviolet radiation.

Ozone has been used in treatment of municipal wastewater effluents primary for disinfection. However, its powerful capacities to react with organic matter in wastewater led to another application of ozone in water treatment. Beside disinfection, ozonation can improve the general physical and chemical quality of effluents, such as reducing chemical oxygen demand (COD), biological oxygen demand ($BOD_{5)}$, color, and UV absorbance, and increasing dissolved oxygen (DO) [6]. Because of its high oxidation potential, ozone reacts with a wide range of organic and inorganic compounds in water. Chemical oxidation by ozone occurs by two distinct reaction mechanisms, namely a molecular ozone reaction pathway and a hydroxyl radical ($^{\bullet}OH$)

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reaction mechanism. While ozone is a very selective oxidant which reacts quickly with double bonds, activated aromatic compounds and deprotonated amines, OH^- radicals react with most water constituents with nearly diffusion controlled rates. Many previous studies of ozonation indicated that ozone attacks aromatic and unsaturated compounds, thereby affecting the chemical composition and the overall quality of the water [7].

However, at the low level of ozone dose used in wastewater treatment plant, the total organic carbon is not significantly affected [5]. The limited dissolved organic carbon removal that was observed in the ozonation stage shows that oxidation leads to form the transformation products rather than mineralization [8]. The ozonated organic compound is readily biodegradable the original compounds. So ozone oxidation is a promising process as a supplementary method for biological treatment [9].

Algae *Chlorella*sp. was widely applied for wastewater treatment and had proven abilities of removing nitrogen, phosphorus, and chemical oxygen demand (COD) with different retention times ranging from 10 h to 42 days, mixing with bacteria or not, which shows the potential of replacing activated sludge process in a secondary or tertiary step in view of nutrient reduction and biomass production [11].

Moreover, astaxanthin (3,3'-dihydroxy-β,β-carotene-4,4'-dione), a product of *Chlorella zofingiensis's* metabolism with a high-value ketocarotenoid with a broad range of applications in food, feed, nutraceutical, and pharmaceutical industries, has been gaining great attention from science and community in recent years. The green microalgae *Chlorella zofingiensis* represents the most promising producers of natural astaxanthin. *C. zofingiensis* grows fast phototrophically, heterotrophically and mixtrophically, is easy to be cultured and scaled up both indoors and outdoors, and can achieve ultrahigh cell densities. These robust biotechnological traits provide *C. zofingiensis* with high potential for mass astaxanthin production [4].

From this point of view, we study the combination of ozonation and heterotrophic cultivation of *Cholorella zofingiensis* in domestic wastewater treatment and astaxanthin production.

2. Materials and methods

2.1. Materials

Raw wastewater samples were collected from the Koto Domestic Wastewater Treatment Plant (Okayama city, Japan). In every sampling, wastewater was taken at the influent to the primary sedimentation tank which was taken from 2014, July 4thuntil 2015, January 29th. The experiment was conducted at natural pH conditions of wastewater; pH is close to it.

Ozone was applied to the coagulation supernatant; the supernatant was derived after coagulation and sedimentation of the wastewater with coagulation conditions of 4 mg/L chitosan dosage without pH adjustment. Ozone was generated by Mitsubishi OS-1N ozonizer.

2.2. Methods

Ozonation experiment was conducted at 3 different electricity occurrence of Ozonize, which would produce different ozone doses. The *Chlorella zofingiensis* was cultivated in

250mL of ozonated wastewater. The cultivation is continued in three weeks.The culture conditions are shown in table 1.

***Table 1**. Culture conditions of C. zofingiensis in ozonatedwastwater*

Valuables	Conditions
Ozone consumption	0; 2.6; 4.33; 5.72mg O_3 L^{-1}
Temperature	27°C

(1) Daily sampling.

A 3mL of sample was taken from every flask every day for measurement of turbidity

(2) Sampling at every 3 days.

A 10mL of sample was taken from every flask every 3 days. The samples were to be filtered with glass microfiber filter GF/B. The filters were to be used for the measurement of suspended solid, while the filtrates are subjected for the measurement of dissolved nitrogen (DN), dissolved phosphorus (DP), dissolved organic carbon (DOC).

(3) Sampling on the final day.

A 5 mL of samples is subjected for the measurement of astaxanthin. The remaining in the flasks is used for the measurement for the items mentioned in (2).

Analytical methods

In order to determine the physical-chemical characteristics of the effluents and treated effluents, a large number of analyses based on Standard Methods for the Examination of Water and Wastewater (APHA, 2005) were conducted on each sample and the following parameters were measured: pH, Zeta Potential, Turbidity, Total Organic Carbon (TOC), Total Phosphorus (TP), Total Nitrogen (TN).

3. Results and discussions

3.1. Ozonation

Raw wastewater was taken from Koto Domestic Wastewater Treatment Plant, Okayama city, Japan. The characteristics of sample are shown in table 2.

***Table 2**. Characteristics of raw wastewater*

Parameters	Average	Range
pH	6.93	6.46 - 7.2
Turbidity (Absorbance at 660nm)	0.10	0.031 - 0.18
Zeta potential	-17.95	-20.5 - -14.1
UV254	0.60	0.131- 1.087
Total Nitrogen (mgL^{-1})	26.10	11.83 - 37.60
Total Phosphorus (mgL^{-1})	5.22	1.19 - 11.19
Total Organic Carbon (mgL^{-1})	27.38	4.724 - 47.57

Before conducting ozonation to waste water, seawage was removed solid, turbidity by chitosan coagulation without pH adjustment. The properties of coagulated wastewater are shown in table 3.

***Table 3**. Characteristics of coagulated wastewater*

Parameters	Average	Range	Removal rate (%)
pH	6.91	6.64 - 7.18	-
Turbidity (Absorbance at 660nm)	0.02	0.009 ≈ 0.045	74.30
Zeta potential	-15.55	-19.4 ≈ -1.61	-
UV254	0.31	0.083 ≈ 0.717	45.66
Total Nitrogen (mgL^{-1})	23.30	11.18 ≈ 36.03	11.57
Total Phosphorus (mgL^{-1})	3.76	0.56 ≈ 5.94	21.68
Total Organic Carbon (mgL^{-1})	19.12	1.83 ≈ 30.7	39.16

3.2. Effects of ozonation on total organic carbon of coagulated wastewater

In this study, ozonation was applied to coagulated wastewater at 4mg/L of chitosan of domestic wastewater taken weekly from December 12th, 2014 to January 29th, 2015.

From the results shown in Figure 3.1, the total organic carbon was hardly removed, but increased. This result is similar to that Asano's work [1]. The TOC removals were varied, and no apparent trend was observed. In the ozonation process, to observe the TOC level reduction, a significant part of the organic carbon must be completely oxidized to CO_2. However, the ozone dose examined in this study was low, and consequently it is considered unlikely that any significant degree of the complete oxidation occurred.

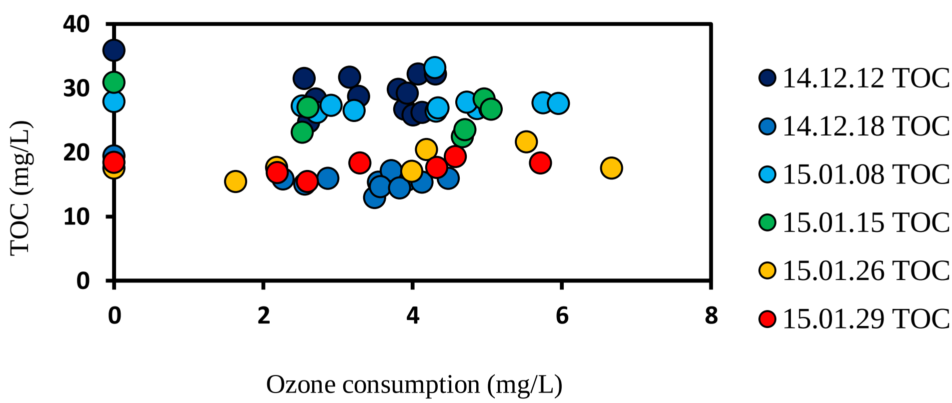


***Figure 1.** Total organic carbon profiles of different wastewater samples at diffirent ozonation conditions*

The value of the UV absorbance at 254 nm is indicative of organic species having double bonds and an aromatic structure. The reduction of this parameter is consistent with established reaction mechanisms whereby molecular ozone readily reacts with both unsaturated and aromatic compounds [6]. SUVA- specific ultraviolet light absorbance, an indicator of the aromaticity of organic matter in water, was defined by UV254 divided by TOC.

Figure 2 shows the change of SUVA during the treatment of domestic wastewaters with different ozone doses. This value could provide insights into the characteristics of water such as aromatic contents per unit concentration of organic carbon, hydrophobicity, and molecular weight distribution of DOC. Unlike TOC, the SUVA value of wastewater were decreased with the increasing of ozone consumptions , indicating that may be occur destruction of unsaturated bonds in organic matter. According to previous studies, limited reduction of DOC but dramatic decrease in SUVA was used as an evidence for the destruction of the unsaturated bonds by ozonation [10].

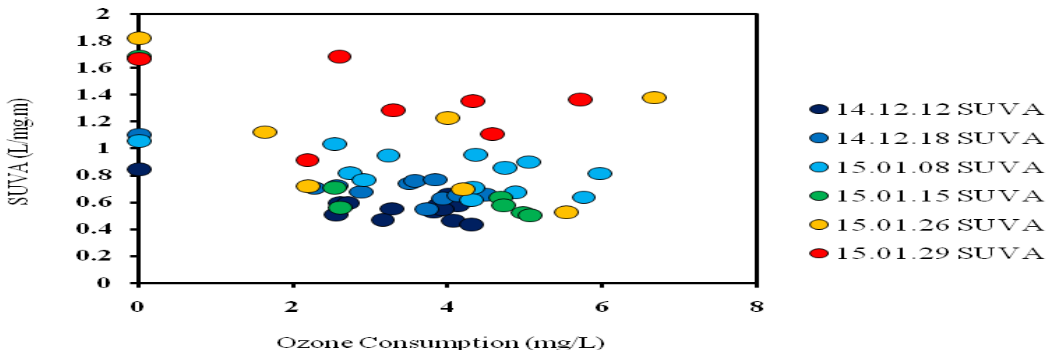


Figure 2. *SUVA of different wastewater samples at different ozonation treatment condition*

In this research, the water tested had a relatively low SUVA value (1.87±1.09 L/mg-m), indicating that the water contained hydrophilic and low-molecular-weight materials. SUVA decreased with increasing ozone consumption. This means that ozone processes could alter hydrophobic to hydrophilic and high-molecular-weight to low-molecular-weight organic matter.

From this result, it was confirmed that low concentrated ozone had a limitation of organic oxidation due to the selective reaction and the partial oxidation with organics by ozone. The ozone process alone could be proposed as a pre-treatment for the biological treatment because ozonation was able to enhance the biodegradability in the water.

3.3. Cultivation of Chlorella zofingiensis

To evaluate effects of ozonation on the growth of *Chlorella zofingiensis*, the algae were cultivated in ozonated wastewaters with different ozone dosages. The ozonated wastewater was prepared by ozonation of coagulated wastewater taken on January 29th, 2015.

In heterotrophic cultivation, *C. zofingiensis* used dissolved organic matters as carbon source. The concentration and component of organic carbon would affect *C. zofingiensis'* growth.

In this experiment, the carbon source for *C. zofingiensis*is dissolved organic carbon in wastewater. Especially the organic carbons have transform from high-molecular-weight to low-molecular-weight compounds. The dissolved organic carbon of ozonated wastewater was presented in table 4.

Table 4. *Dissolved organic matter of ozonated wastewaters*

Ozone dosage (mgO_3/L)	Dissolved Organic Carbon (mg/L)
0	20.94
2.60	19.97
4.33	20.03
5.72	21.56

The figure 3 shows the growth of *Chlorella zofingiensis* in different ozonated wastewaters. The algae adapted well and grown fast in 3 first days of cultivation. The algae exhibited the highest specific growth rate at day 3 of cultivation with 0.084; 0.076; 0.151; 0.061 (day^{-1}) in different ozonated wastewaters; and the highest biomass are 0.006; 0.005;

0.012; and 0.003 mg/day. Microalgae also uptakes dissolved organic carbon, nitrogen and phosphorus compounds for growing. Therefore DOC, DN, and DP are removed in wastewater. The nutrient removal of *C. zofingiensis* is shown in figure 4. The DOC removed with the highest rate at day 6 of cultivation. The DOC removal rates in ozonated wastewaters were higher than non-ozonated wastewater, 47.26% - 53.47% to 35.55%; and the highest removal rates of DOC were 53.47% for 4.33 mgO_3/L ozonated wastewater.

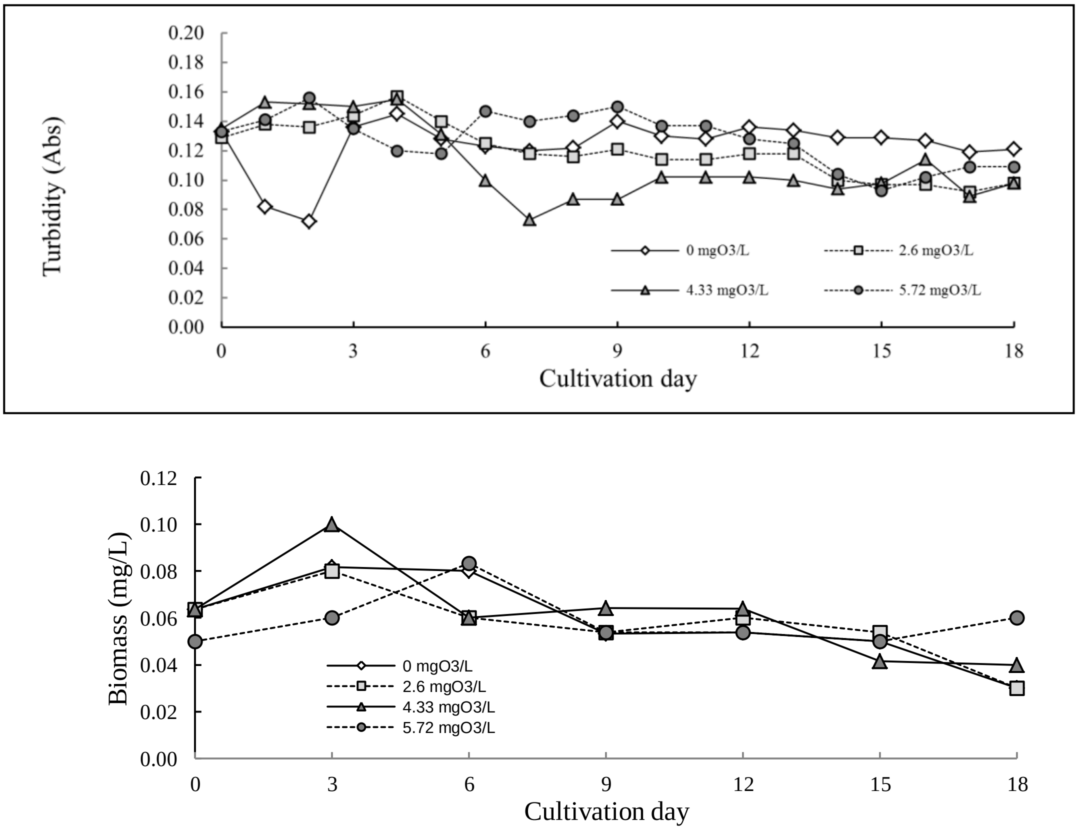
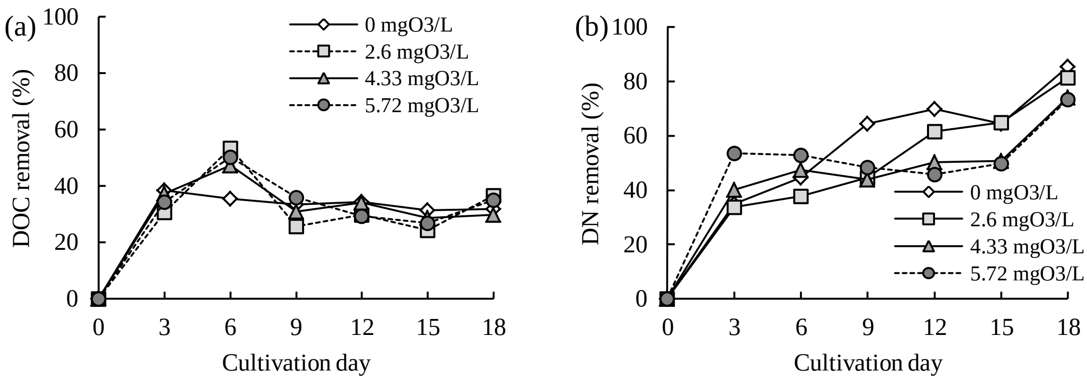


Figure 3. *The growth (a) and biomass (b) of Chlorella zofingiensis when cultivated in different ozonated wastewaters*

On day 6 of cultivation, the DP of wastewater were removed with 34.51%; 32.12%; 33.02%, and 9.98% removal ratios for 0 ; 2.6 ; 4.33; and 5.72 mgO_3/L, respectively. The removal ratios of DP in ozonated wastewater with 4.33 and 5.72 mgO_3/L were higher in the day 9 of cultivation but not significant (39.7 and 12.0% respectively). Unlike the DOC and DP, DN of wastewater continued to be removed even the algal growth and biomass decreased. On day 6 of cultivation, the DN removal ratios were 37-52%. And at the final day of cultivation, DN removals for different ozonated wastewaters increased to 85.34; 81.33; 74.13; and 73.46%, respectively.

Astaxanthin production was also objective of heterotrophic *Chlorella zofingiensis* cultivation. The result is shown in figure 5. The astaxanthin production increased with the increasing of ozone consumption. The highest production was 0.009 $mg.L^{-1}$ when algae cultivated in 5.72 $mgO_3.L^{-1}$ wastewater. However, the astaxanthin productions were low, may be due to the low concentration of DOC in wastewaters.



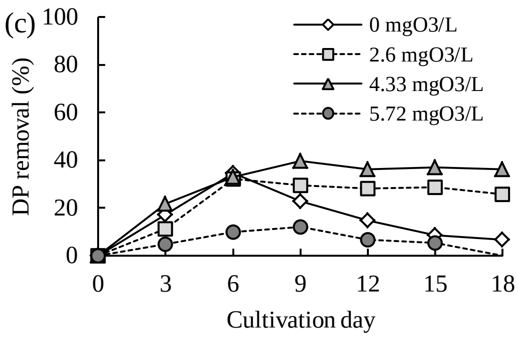


Figure 4. *Nutrient removal capacity of Chlorella zofingiensis when cultivated in diferrent ozonated wastewaters. a, Dissolved organic carbon removal rate; b, Dissolved nitrogen removal rate; c, Dissolved phosphorous removal rate.*

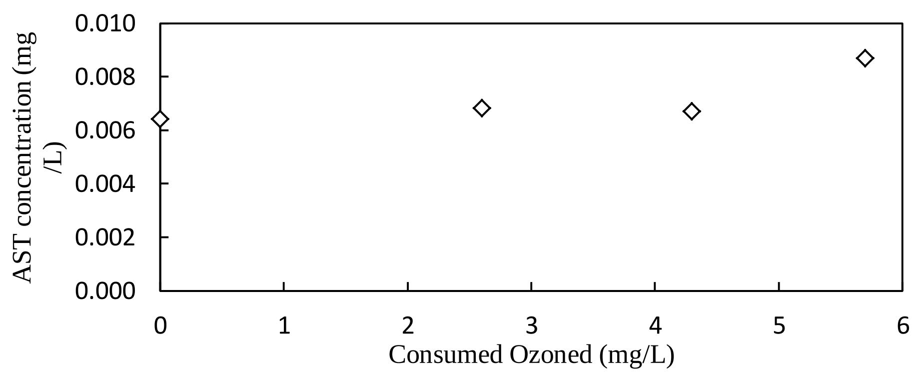


Figure 5. *The astaxanthinproduction of Chlorella zofingiensis when cultivated in different ozonated wastewater after 18 days of cultivation. (This result was obtained with the help of Hirotaka Komatsu)*

4. Conclusion

Ozonation not only removed but produced transformation of organic matters; also increased capacity of dissolved organic carbon and dissolved nitrogen removal of *Chlorella zofingiensis* as well. On day 6 of *Chlorella zofingiensis* cultivation, dissolved organic carbon removal ratios in ozonated wastewaters were higher than non-ozonated wastewater, with 47.26% - 53.47% to 35.55%. And the dissolved nitrogen removal ratios were 47.38 and 52.90% if 4.33 and 5.72 mgO_3/L applied compared to 44.59% of non-ozonated wastewater. The astaxanthin production of *Chlorella zofingiensis* in heterotrophic cultivation was the highest (0.009 mg/L) when cultivated in ozonated wastewater with the highest ozone dose applied (5.72 mgO_3/L). Therefore, the ozonation - heterotrophic cultivation of *Chlorella zofingiensis* process could be used in domestic wastewater treatment. However, as dual-goal of algae cultivation, astaxanthin production in this study is not effective.

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VEGETABLE FARMERS PERCEPTION OF PESTICIDE USE PRACTICES IN THANH HOA PROVINCE, VIETNAM

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Received: 12 February 2019/ Accepted: 11 June 2019/ Published: June 2019

Abstract: *Pesticide use practices among smallholder vegetable farmers in Thanh Hoa province were investigated through field surveys, questionnaires, and interviews. This study was carried out to assess the knowledge and perception of vegetable growers regarding the appropriate knowledge on safe handling and proper use of pesticides. The results revealed that farmers' choices of pesticides was strongly influenced by both authorized pesticide dealers and neighboring farmers. In addition, Over 87% farmers did not read written information on pesticide label before use. Consequently, most farmers applied pesticides in violation of the recommendations: they overused, misused, and abused pesticides for pest control, ignore risks and safety instructions, they did not respect pre-harvest interval and dispose containers unsafely. Improved safety training and provided further information on technical aspects of pesticide use would be a cost-effective solution to raise awareness of vegetable farmers about negative impact of pesticides on population health and the environment.*

Keywords: *Pesticide use, pesticide application, environment, agriculture, Thanh Hoa province.*

1. Introduction

Pesticides are widely used in agricultural production to control insects, diseases, weeds and other undesirable pests, thus constitute one of the most important inputs in crop production [3], [5], [12]. However, the increased use and misuse of pesticides have negative health effects on farmers, traders and consumers, and threaten the natural environment. Unsafe and indiscriminate use of pesticides is common in tropical agricultural systems of developing countries including Vietnam due to farmers poor knowledge on the hazards of pesticide use, risks of hazardous agrochemicals and ineffective governmental enforcement of pesticides' regulations [2], [6]. There are about 80% of Vietnamese farmers using pesticides incorrectly (i.e. violating the '4Rright' principles) causing environmental damage and a number of human health effects [11]. Besides, the types of pesticides and the active ingredients (AI) of pesticides of toxic categories II (moderately hazardous), III (slightly hazardous), and U (unlikely to present acute hazard) have increased considerably (7.4-, 5.9- and 9.1-fold, respectively) from 2002 to 2013 in Vietnamese pesticide market [1]. Recently, low-quality pesticides and counterfeit pesticide is also a major problem for farmers, food

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consumers and the environment. Consequently, more than 7000 cases of pesticides residue poisoning were reported in 2002 in Vietnam [10].

In Thanh Hoa province, where the people living by agriculture account for 85 % of the population, the production areas of vegetable had expanded remarkably to 38762 ha by 2014, leading to a rapidly increase of total quantity of pesticides used for managing insects, weeds and diseases. 110 tone of pesticides with 420 different types of pesticides were applied in field to protect crop production in 2015. It has been shown that more and more local vegetable farmers are now applying pesticides intensively to their crops and relying heavily on the use of pesticides in order to improve vegetable production to meet local market and export market demand. These might pose threats to population health, vegetable consumers, and the environment. Thus, the purpose of this study is to investigate into pesticide knowledge, attitudes related the use of pesticide, safety practice pesticides among vegetable farmers in Thanh Hoa province. The study provided information about pesticide use practices including the types of pesticides used, factors that influence pesticide selection and use, ability to read and understand information written on the label and pesticide safety training. This information will be useful for pesticide policy enforcement and development of more sustainable pesticides use practices by vegetable farmers.

2. Material and methods

Selection method of research area: Interviews and surveys were conducted in Thanh Hoa province, which is located in the North Central Coast region of Vietnam, in December 2017. We selected 450 vegetable farmers for interviews from 5 districts which represent 3 specific areas, includes: Thieu Hoa and Thanh Hoa city represent the dental region; Hoang Hoa, Tinh Gia represents the coastal region and Tho Xuan represents the mountainous region.

Methods of collecting information: Primary data collection: qualitative and quantitative data were collected from vegetable growers through farm survey by face-to-face interviews with vegetable farmers/sprayers, in-depth interviews managers, group discussions Participatory Rural Appraisal (PRA) and field observations. A questionnaire containing structured closed-ended questions was designed based on relevant published literature. Secondary data were collected from the legislation and the provisions of plant protection in Thanh Hoa province. Data were analysed by Microsoft EXCEL and Statistical Package for Social Sciences (SPSS) software 22.0.

3. Results

Pesticide use and safety training: The result of table 1 indicates that vegetable crops area is approximately 38432 ha in 2015 in Thanh Hoa province. Pesticide consumption per year continuously reduced by 110 tons in 2015. Likewise, pesticide use per hectare also decreased by 0.27 kg/ha in 2015. However, the number of cases of pesticide poisoning incidents did not significantly reduce from 2013 to 2015 suggesting the main reason causing pesticide poisoning to farmers and consumers in Thanh Hoa province in 2015 which possibly related to pesticide use practices (Table 1). In addition, 25.11% of vegetable growers reported that they did not received any training or technical support on safe pesticide use. Besides, the safety training frequency for farmers differed significantly among regions. 18.33 % of vegetable growers in delta areas (n =180) received frequently pesticide safety training while the percentage of farmers coastal areas and attended frequently pesticide safety training were 5% and 4.44% respectively (table 2). In terms of training content, approximately 50% of respondents participated in pesticide products, which mainly focused on advertising and selling pesticides products, but not pesticide use practice (table 2).

Table 1. *The 2013-2015 report on Pesticide use in Thanh Hoa*

Item	Unit	2013	2014	2015
1. Annual crop area	ha	447102	448928	443680
Rice	ha	256300	258600	257000
Vegetable	ha	38100	38762	38432
2. Pesticide consumption	ton	290	146	110
3. Pesticide use per hectare	kg/ha	0,648	0,325	0,247
4. Pesticide poisoning incidents	Number of cases	17	14	15

Table 2. *Training received by farmers on safe pesticide use in studied locations*

Unit: (%) percentage of household

Variable	Delta areas (n=180)	Coastal areas (n=180)	Inland areas (n=90)	Mean (n=450)
1. Safety training frequency				
Frequently	18,33	5,00	4,44	10,22
Sometimes	80,56	63,33	35,56	64,67
Not yet	1,11	31,67	60,00	25,11
2. Training contents				
Pesticide products	45,56	51,67	56,67	50,22
Pesticide use	54,44	48,33	43,33	49,78

Pesticide use practices: The list of pesticides used by farmers in surveyed locations is showed in Supplementary Table 1. The data show that insecticides (61.81%) are the most used pesticides, follows by fungicides (30.9%) and herbicides (7.2%) usage. There were no pesticides classified as extremely hazardous (Ia) or highly hazardous (Ib) being applied based on the WHO guidelines to classification of pesticides by hazard (2009). However, 28 out of 55 (50.9 %) of pesticides were unregistered for use on vegetables. Moreover, 17 out of 55 (30.9 %) of pesticides classified as moderately hazardous (II) were widely applied in the vegetable production area (Supplementary Table 1) because they found them very effective.

Supplementary Table 2 presents the fact of pesticide use practices of vegetable growers in surveyed locations. 33.78% of vegetable farmers selected pesticides according to neighbour's recommendation. Over 60% of farmers were directly influenced by authorized dealer recommendations, while 30.89% followed extension worker's recommendation. For pesticide application timing, spraying in the afternoon was the most common (45.33%). 10.44% of farmers sprayed pesticide in early morning while 8.44% of those applied pesticide at any time of the day. Most farmers applied pesticide was based on noticing crop damage

(43.33%) and neighbour's recommendation (45.56%). Few (11.11%) farmers followed extension worker's recommendation to apply pesticide at right time in their crop. 80.22% of farmers interviewed in surveyed areas applied pesticide at higher rate (from 1.5 to 2 times) than permitted by the label on the pesticide product. They assumed that applying pesticide at higher dose would achieve greater effectiveness to control pests and diseases in their crop, without considering the effects on their health, vegetable consumers and the environment.

For pesticide spraying techniques in the field, the results of Supplementary Table 2 shows that 24.67% of farmers sprayed pesticides with the wind direction to minimize their exposure to toxic pesticide, while 6.22% did not consider wind direction when spraying pesticide. Over 87% of the farmers did not read written information on pesticide label before use, including the direction on how to mix, apply in the field, because they were unable to read and understand the meaning of the label. Few (12.44%) farmers read and understood pesticide labels correctly. Besides, often two or more pesticides were mixed together in the sprayer tank without consideration of pesticide compatibility or effects on workers. Those famers reasoned that mixtures would result in higher effectiveness of pest control and control more than one pest with the same application. According to Ngowi et al. (2007), mixing more than two different types of pesticides possibly causes interactions between fungicides, insecticides and water mineral content, resulting in reduction of pesticide efficacy, the mixture could be less effective to pests, more toxic to sprayers and the environment.

For pre-harvest interval (PHI), 51.78% of interviewed vegetable growers still harvested vegetables before the pre-harvest interval written on the label expires due to their economic profit, 7.78 % of respondents did not check the label of pesticide products for pre-harvest interval. For protective measures during spraying, 90.44% of respondents used mask, gloves (11.78%), boots (25.33%), hat (69.11%), and raincoat or safety clothes (7.11%).

Farmers' attitudes toward effect of pesticides on the environment: The most common way of disposing of leftover pesticides were spraying until no pesticide left (91.56%). For disposal empty pesticide containers after use, 48.44% of vegetable growers reported to throw them in the field. Alternatively, 47.78% gathered and kept in safe places. Most farmers (86.89%) cleaned pesticide application equipment after use, including spray tanks, valves, booms, nozzles in the field without concerning about contamination of water by rinse water and the remains of pesticide.

4. Discussion

Pesticide use practices and pesticide knowledge among smallholder vegetable farmers in Thanh Hoa province were surveyed. This study indicated that most of vegetable farmers apply pesticides indiscriminately in violation of the recommendations: they still relied heavily on and overused pesticides, moderate hazardous (II) pesticides were still used widely in the vegetable production area, used unregistered pesticides for use on vegetables, ignore risks and safety instructions and did not use protective devices when applying pesticides. These problems can be possibly attributed to farmers' lack of pesticide knowledge and pesticide safety training. Similarly, the study of Mengistie et al. (2017) revealed that training on safe pesticide use significantly influences on the knowledge, attitudes, and practices concerning pesticide use of vegetable farmers. Lack of technical pesticide knowledge, the absence of extension services and lack of pesticide safety training results in pesticide misuse (abuse and overuse) by farmers. On the other hand, Nguyen et al. (2018) reported that though a high number of vegetable farmers receive training on pesticide use, most of them still violated pesticide recommendations, applied widely moderate hazardous (II) and unregistered pesticides for vegetable production that affect the safety of vegetables for consumption.

Our study shows that the choice of pesticides to be used by vegetable growers was strongly influenced by both authorized pesticide dealers and neighboring farmers. This is also common in developing countries including Ethiopia central rift valley [6]. Farmers generally use a higher dosage of pesticides than recommended in their crops because they want to reduce spraying frequency and eliminate pests at once. Excessive pesticides use may lead to high residue levels on plants, which may be toxic to vegetable consumers and the environment [14]. Mengistie et al. (2017) reported that 87% of famers mix two pesticides without considering undesirable interactions between pesticides in mixture which may lead to reducing active ingredient effectiveness or adverse effects to the pests, damaging their health or the environment [6], [7], [13]. Our survey showed that many vegetable farmers did not use protective equipment such as gloves, boots, hat and safety clothes during spraying due to lack of availability and affordability. The use of protective equipment makes farmers feel uncomfortable under local hot and humid climates and cumbersome during working, while some consider it too expensive to access [9], [6].

Most of farmers in surveyed location did not read the pesticide label carefully before applying. Consequently, they did not follow pre-harvested interval written on the pesticide label or consider pre-harvest interval when they applied pesticides. Another reason mentioned for not respecting recommended pre-harvested interval was economic profit in which harvest time was almost determined by vegetable dealers and market demand. The study of Nguyen et al. (2017) also showed that 98% of farmers were aware of recommended pre-harvested interval written on pesticide label, but they did not always follow it. According to Jeyanthi and Kombairaju (2005), the level of pesticide residues still greatly remained on vegetable products before with holding period.

5. Conclusion

The study revealed that vegetable farmers in surveyed location including 5 districts (Hoang Hoa, Thieu Hoa, Tinh Gia, Tho Xuan and Thanh Hoa city) in Thanh Hoa province apply pesticides indiscriminately in violation of the recommendations. They often overuse, misuse, abuse pesticides for pest control in their crops without considering potential threats to their health, vegetable consumers and the environment. The farmers' choices of pesticides was strongly influenced by both authorized pesticide dealers and neighboring farmers, and most of the farmers did not read written information on pesticide label before use was possibly an important reason for indiscriminate use and improper application of pesticides by vegetable farmers. Providing further information and safety training courses on the economic, scientific, legal and technical aspects of pesticides could be feasible solutions for raising awareness among vegetable growers about potential hazards of pesticides to their health, consumers and the environment.

Supplementary Table 1. *List of pesticides used by farmers in vegetable production in Thanh Hoa province, Vietnam*

Types of Pesticides	Trade name	Active ingredients (Ai)	Registered for use on	Toxic class (by WHO*)	Original pesticides
Insecticide (61.81%)	Acdinosin 50WP	Nitenpyram; Fipronil; Dinotefuran	Rice	III	Synthetic pesticide
	Amico 10EC	Imidacloprid	Rice	III	Synthetic pesticide
	Anvado 100WP	Imidacloprid	Rice	II	Synthetic pesticide
	Bafurit 5WG	Emamectin benzoate	Rice, vegetables, tea	III	Bio-pesticide
	Bemab 52WG	Emamectin benzoate	Rice, vegetables	II	Bio-pesticide
	Bestox 5EC	Alpha_cypermethrin	Rice, soybean	II	Synthetic pesticide
	Binova 45WP	Acetamiprid; Buprofezin	Rice	II	Synthetic pesticide
	Blugent 75SC	Fipronil; Indoxacarb	Rice	II	Synthetic pesticide
	Calira 555WP	Imidacloprid; Acetamiprid; Buprotezin	Rice	III	Synthetic pesticide
	Checsura 500WP	Chlorpyrifos Ethyl; Acetamiprid	Rice	II	Synthetic pesticide
	Conphai 15 WG	Imidacloprid	Rice, coffee	II	Synthetic pesticide
	Dofaben 100WG	Emamectin benzoate	Rice, vegetables	II	Bio-pesticide
	Dylan 2EC	Emamectin benzoate	Rice, vegetables, fruits	III	Synthetic pesticide
	Ema aici 50WG	Emamectin benzoate	Rice, vegetables	III	Bio-pesticide
	Fm-Tox25EC	Alpha cypermethrin	Rice, vegetables, coffee	II	Synthetic pesticide
	Goldra250WG	Thlamethoxam; Acetamiprid	Rice, sugarcane	II	Synthetic pesticide
	Golnitor 50WDG	Emamectin benzoate	Rice, vegetables, fruits	III	Bio-pesticide
	Goltoc 250EC	Quinalphos; Fipronil	Rice	II	Synthetic pesticide

Types of Pesticides	Trade name	Active ingredients (Ai)	Registered for use on	Toxic class (by WHO*)	Original pesticides
	Marshal 200SC	Carbosulfan	Various crops	II	Synthetic pesticide
	Motsuper 36.0WG	Acetamiprid	Rice, vegetables	II	Synthetic pesticide
	Peran 50 EC	permethrin	Rice, vegetables	II	Synthetic pesticide
	Picmec 666 EC	Chlorpyrifos Ethyl; Alpha -Cypermethrin ; Quinalphos	Rice	II	Synthetic pesticide
	Regent 800WP	Fipronil;	Rice	II	Synthetic pesticide
	Rholam Super 50SG	Emamectin benzoate; Matrine	Rice, vegetables	III	Bio-pesticide
	Sieufatoc 36EC	Abamectin; Emamectin benzoate	Rice, vegetables, fruits	II	Synthetic pesticide
	Sokupi 0,5SL	Martrine	Rice, vegetables	IV	Bio-pesticide
	Spaceloft 595EC	Alpha_cypermethrin; Chlorpyrifos Ethyl; Imidacloprid	Various crops	II	Synthetic pesticide
	Scorpion 36EC	Abamectin; Fipronil	Rice, vegetables	II	Synthetic pesticide
	TaSieu 5WG	Emamectin benzoate	Rice, vegetables	III	Synthetic pesticide
	Tomuki 50 EC	Hexy thiazox	Flowers, vegetables	IV	Synthetic pesticide
	Tungent 5SC	Fipronil	Various crops	II	Synthetic pesticide
	Clothion 55EC	Chlorpyrifos Ethy; Cypermethrin	Rice, vegetable	II	Synthetic pesticide
	V.K 16WP	*Bacillus thuringiensis*	Vegatable, cotton	IV	Bio-pesticide
	Virtako 40WG	Cholrantraniliprole; thiamethoxam	Rice, vegetables, maize	IV	Synthetic pesticide
Fungicide (30.9%)	Agofast 80WP	Fosetyl aluminium	Various crops	IV	Synthetic pesticide
	Aliette	Sosetil aluminium	Rice, vegetables	III	Synthetic pesticide
	Carozate 72WP	Mancozeb; cymoxanil	Rice, vegetables	III	Synthetic pesticide

Types of Pesticides	Trade name	Active ingredients (Ai)	Registered for use on	Toxic class (by WHO*)	Original pesticides
	Cythala 75WP	cymoxanil; Chlorothalonil	Rice, water melon	IV	Synthetic pesticide
	Daconil 40WG	Chlorothalonil	Rice, vegetables	III	Synthetic pesticide
	Kasumin 2SL	Kasugamycin	Rice, vegetables, fruits	IV	Synthetic pesticide
	Mexyl MZ 72WP	Metalaxyl; Mancozeb	Rice, fruits	IV	Synthetic pesticide
	Ricide 72WP	Metalaxyl; Mancozeb	Fruits	IV	Synthetic pesticide
	Rido Xanil 750WP	Cymoxanil; Mancozeb	Rice, vegetables	III	Synthetic pesticide
	Score 250EC	Difenoconazole	Vegetables, fruits	III	Synthetic pesticide
	Strepa 150WP	Streptomycinsulfate	Rice, vegetables	IV	Synthetic pesticide
	Topsin M 70WP	Thiophanate Methyl	Rice, vegetables, fruits	III	Synthetic pesticide
	Totan 200 WP	Bronopol	Rice	III	Synthetic pesticide
	Validan 3SL	Validamycin A	Rice, vegetables	IV	Synthetic pesticide
	Validacin 5L	Validamycin A	Rice, vegetables	IV	Synthetic pesticide
	ZIMVIL 720 WP	MEtalaxyl; Mancozeb	Rice, vegetables, fruits	IV	Synthetic pesticide
	Zithane Z 80 WP	Zinneb	Tomato, grape	III	Synthetic pesticide
Herbicide (7.2%)	Fansipan 200SL	Paraquat ion	Various crops	II	Synthetic pesticide
	Gfaxone 20 SL	Paraquat ion	Various crops	II	Synthetic pesticide
	Power up 275 SL	Paraquat Dichloride	Rice	II	Synthetic pesticide
	Vocal 276 SL	Paraquat Dichloride	Various crops	II	Synthetic pesticide

**WHO: World Health Organization; II – moderately hazardous; III – slightly hazardous; IV-unlikely to present acute hazard in normal use.*

Supplementary Table 2. *Pesticide use practices of vegetable growers in studied locations*

Unit: (%) percentage of household

Survey question	**Delta areas (n=180)**	**Coastal areas (n=180)**	**Inland areas (n=90)**	**Mean (n=450)**
1. Selecting a pesticide				
Extension workers' recommendation	34,44	27,78	30,00	30,89
Neighbours' recommendation	36,11	18,89	58,89	33,78
Authorized dealer	56,67	63,33	76,67	63,33
Personal experience	78,89	45,00	53,33	60,22
2. Pesticide application timing				
Early morning	13,33	11,67	2,22	10,44
Moring	36,11	35,00	36,67	35,78
Afternoon	41,11	44,44	55,56	45,33
Other	9,44	8,89	5,56	8,44
3. Decision to apply pesticide				
Extension worker's recommendation	13,33	10,00	8,89	11,11
Noticing crop damage	39,44	41,67	54,44	43,33
Neighbour's recommendation	47,22	48,33	36,67	45,56
4. Apply pesticide at higher rate than permitted by the label				
Yes	77,78	81,01	83,33	80,22
No	22,22	18,99	16,67	19,78
5. Pesticide spraying techniques in the field				
Spray with the wind direction	27,22	23,33	22,22	24,67
Spray with zig zag model	69,44	68,33	70,00	69,11
Other	3,33	8,33	7,78	6,22
6. Read pesticide label carefully before mixing and applying				
Yes	21,11	6,11	7,78	12,44
No	78,89	93,89	92,22	87,56
7. Respect the recommended pre-harvested interval written on package label				
unnoticed	6,67	8,33	8,89	7,78
No	47,78	51,11	61,11	51,78
Yes	45,56	40,56	30,00	40,45
8. Protective measures during spraying				
8.1. Mask				
Never	1,11	0,00	0,00	0,44

Rarely	2,22	2,78	2,22	2,44
Occasionally	6,11	6,11	8,89	6,67
Regularly	90,56	91,11	88,89	90,44
8.2. Gloves				
Never	22,22	21,11	16,67	20,67
Rarely	30,56	31,11	25,56	29,78
Occasionally	39,44	33,89	42,22	37,78
Regularly	7,78	13,89	15,56	11,78
8.3. Boots				
Never	2,78	21,11	12,22	12,00
Rarely	10,00	50,00	23,33	28,67
Occasionally	34,44	25,56	50,00	34,00
Regularly	52,78	3,33	14,44	25,33
8.4. Hat				
Never	7,22	5,56	3,33	5,78
Rarely	10,00	7,78	5,56	8,22
Occasionally	20,00	16,11	12,22	16,89
Regularly	62,78	70,56	78,89	69,11
8.5. Raincoat or safety clothes				
Never	21,11	24,44	26,67	23,56
Rarely	50,00	43,33	42,22	45,78
Occasionally	21,11	23,33	28,89	23,56
Regularly	7,78	8,89	2,22	7,11
9. Leftover pesticides				
Spray until no pesticide left	88,89	94,44	91,11	91,56
Dump in the field	8,89	5,56	1,11	6,00
Spray other crops	2,22	0,00	7,78	2,44
10. Disposal empty pesticide containers are				
Kept in safe place	62,78	38,89	35,56	47,78
Left in field	37,22	55,56	56,67	48,44
other	0,00	5,56	7,78	3,78
11. Cleaning pesticide application equipment				
In field	75,00	97,22	90,00	86,89
In safe place	21,11	2,78	6,67	10,89
House	3,89	0,00	3,33	2,22

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NOVEL EXPONENTIAL STABILITY CRITERION OF NONLINEAR SYSTEMS WITH INTERVAL TIME-VARYING DELAYS

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Received: 18 January 2019/ Accepted: 11 June 2019/ Published: June 2019

Abstract: *This paper presents a new result on delay-dependent exponential stability for nonlinear linear systems with interval time-varying delay. By constructing a set of improved Lyapunov-Krasovskii functionals combined with Wirtinger-based integral inequality, a new delay-dependent condition is established in terms of linear matrix inequality (LMI) which guarantees that the system is exponential stability.*

Keywords: *Exponential stability, nonlinear systems, Wirtinger-based integral inequality, Interval time-varying delays.*

1. Introduction

In the scope of functional differential equations, stability problem has been the subject of investigable research attention. Among the well-known Lyapunov stability method, the Lyapunov functional is a powerful tool for stability analysis of time-delay systems [2], [3], [7], [10]. Based on the Lyapunov function, delay-dependent stability criteria for these systems are established in terms of linear matrix inequalities (LMIs). On the other hand, the exponential stability problem for differential systems has received the attention of many mathematicians in recent times. However, to the best our knowledge, the problem of exponential stability differential systems with state delays has not been fully investigated to date, especially for nonlinear systems with time-varying. The stability criteria have mainly been given for linear systems with constant delay, linear system with time-varying delay [4], [8], [12], [11]. There are very few results about exponential stability for nonlinear systems with time-varying delay. In this research, we have considered the exponential stability problem for a class of nonlinear system with time-varying delays. Based on an improved Lyapunov-Krasovskii functional combined with Wirtinger-based integral inequality, the sufficient condition for the exponential stability for nonlinear systems has been derived in term of LMIs.

Notations: The following notations will be used throughout this paper. R^+ denotes the set of all nonnegative real numbers; R^n denotes the $n-$dimensional Euclidean space with the norm $\|.\|$ and scalar product $\langle x,y\rangle = x^T y$ of two vectors x,y; $\lambda_{max}(A)$ ($\lambda_{min}(A)$, resp.) denotes the maximal (the minimal, resp.) number of the real part of eigenvalues of A; A^T denotes the transpose of the matrix A and I denotes the identity matrix; $Q \geq 0$ ($Q > 0$, resp.) means that Q is semi-positive definite (positive definite, resp.) i.e. $x^T Qx \geq 0$ for all

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$x \in R^n$ (resp. $x^{\mathrm{T}}Qx, > 0$ for all $x \neq 0$); $A \geq B$ means $A - B \geq 0$; $\mathcal{C}^1([-\tau, 0], R^n)$ denotes the set of R^n- valued continuous functions on $[-\tau, 0]$ with the nor $\|\varphi\|_\tau = \max\{\sup_{-\tau \leq t \leq 0} \|\varphi(t)\|, \sup_{-\tau \leq t \leq 0} \|\dot{\varphi}(t)\|\}$.

The segment of the trajectory $x(t)$ is denoted by $x_t = \{x(t+s) : s \in [-\tau, 0]\}$ with its norm $\|x_t\| = \sup_{s \in [-\tau, 0]} \|x(t+s)\|$.

2. Preliminaries and problem statement

Consider the following system with mixed time varying delays

$$\begin{cases} \dot{x}(t) = & Ax(t) + Bx(t-h(t)) + Hg(t, x(t), x(t-h(t))), \quad t \geq 0, \\ x(t) = & \varphi(t), \quad t \in [-\tau, 0], \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the system state; $A, B, H \in R^{n \times n}$ are real known system matrices with appropriate dimensions; The time varying delays $h(t), d(t)$ are continuous functions satisfying $0 < h_1 \leq h(t) \leq h_2$ and $\dot{h}(t) \leq \beta$ where h_1, h_2 are lower and upper bounds of the time varying delays $h(t)$. $\varphi(t) \in \mathcal{C}([-h_2, 0], R^n)$ is the compatible initial function specifying the intial state system. The nonlinear functions $g : R^+ \times R^n \times R^n \to R^n$ satisfies

$$g^T(.)g(.) \leq a^2 x^T(t)E^T Ex(t) + b^2 x^T(t-h(t))F^T Fx(t-h(t)) \tag{2}$$

where E, F are symmetric positive definite matrices and a, b are any real numbers.

Definition 1. *System (1) is said to be $\alpha-$ exponentially stable for $\alpha > 0$ if there exist $N > 0$ such that, for any compatible initial conditions $\varphi(t)$ the solution $x(t, \varphi)$ satisfies*

$$\|x(t, \varphi)\| \leq N \|\varphi\|_\tau e^{-\alpha t}, \quad \forall t \geq 0.$$

We introduce the following technical well-known propositions and lemma, which will be used in the proof of our results.

Proposition 1. *(Matrix Cauchy inequality [5]) For any $M, N \in R^{n \times n}$, $M = M^{\mathrm{T}} > 0$ and $x, y \in R^n$ then $2x^{\mathrm{T}}Ny \leq x^{\mathrm{T}}Mx + y^{\mathrm{T}}N^{\mathrm{T}}M^{-1}Ny$.*

Proposition 2. *([13]) any symmetric positive definite matrix M, scalar $\nu > 0$ and vector function $\omega : [0, \nu] \to R^n$ such that the integrals concerned are well defined, then*

$$\left(\int_0^\nu \omega(s)ds\right)^{\mathrm{T}} M \left(\int_0^\nu \omega(s)ds\right) \leq \nu \int_0^\nu \omega^{\mathrm{T}}(s) M \omega(s) ds.$$

Proposition 3. *(Schur complement Lemma [5]) For given matrices X, Y, Z with appropriate dimensions satisfying $X = X^{\mathrm{T}}, Y = Y^{\mathrm{T}} > 0$.*

Then $\begin{bmatrix} X & Z^{\mathrm{T}} \\ Z & -Y \end{bmatrix} < 0$

if and only if $X + Z^{\mathrm{T}}Y^{-1}Z < 0$.

Proposition 4. *(Wirtinger-based integral inequality [15]) For a give* $n\times n$ *matrix* $W>0$ *and a function* $w:[a,b]\to R^n$ *whose derivative* $\dot{w}\in PC([a,b],R^n)$, *the following inequality holds* $\int_a^b \dot{w}^{\mathrm{T}}(s)W\dot{w}(s)ds \ge \frac{1}{b-a}\xi^{\mathrm{T}}\mathcal{W}\xi$ (3)

where $\mathcal{W}=diag\{W,3W\}$ and $\xi = col\{w(b)-w(a), w(b)+w(a)-\frac{2}{b-a}\int_a^b w(s)ds\}$.

3. Main results

In this section, we propose new conditions ensuring the regularity, impulse free and exponential stability of system (1) as presented in the following theorem.

Firstly, given $\alpha>0$. We denote:

$$\Phi_1(t)=x(t),\quad \Phi_2(t)=x(t-h(t)),\quad \Phi_3(t)=x(t-h_1),\quad \Phi_4(t)=x(t-h_2),\ h_{12}=h_2-h_1,$$

$$\Phi_5(t)=\dot{x}(t),\quad \Phi_6(t)=\Big(\frac{1}{h_2-h(t)}\int_{t-h_2}^{t-h(t)}x(s)ds\Big),\quad \Phi_7(t)=\Big(\frac{1}{h(t)-h_1}\int_{t-h(t)}^{t-h_1}x(s)ds\Big),$$

$$\Phi_8(t)=g(t,x(t),x(t-h(t))),\ \Upsilon(t)=\begin{bmatrix}\Phi_1^{\mathrm{T}}(t) & \Phi_2^{\mathrm{T}}(t) & \Phi_3^{\mathrm{T}}(t) & \Phi_4^{\mathrm{T}}(t) & \Phi_5^{\mathrm{T}}(t) & \Phi_6^{\mathrm{T}}(t) & \Phi_7^{\mathrm{T}}(t)\end{bmatrix}^{\mathrm{T}},$$

$$\Pi(1,1)=PA+A^{\mathrm{T}}P^{\mathrm{T}}+Q+2\alpha P+\varepsilon a^2E^{\mathrm{T}}E-e^{-2\alpha h_1}W_1-e^{-2\alpha h_2}W_2,$$

$$\Pi(1,2)=PB,$$

$$\Pi(1,5)=A^{\mathrm{T}}(h_1^2W_1+h_2^2W_2+h_{12}^2W),$$

$$\Pi(1,8)=PH,$$

$$\Pi(2,2)=-(1-\beta)e^{-2\alpha h_2}Q-8e^{-2\alpha h_2}W+\varepsilon b^2F^{\mathrm{T}}F,$$

$$\Pi(2,3)=-2e^{-2\alpha h_2}W,$$

$$\Pi(2,4)=-2e^{-2\alpha h_2}W,$$

$$\Pi(2,5)=B^{\mathrm{T}}(h_1^2W_1+h_2^2W_2+h_{12}^2W),$$

$$\Pi(2,6)=6e^{-2\alpha h_2}W,$$

$$\Pi(2,7)=6e^{-2\alpha h_2}W,$$

$$\Pi(3,3)=e^{-2\alpha h_1}Z-e^{-2\alpha h_1}W_1-4e^{-2\alpha h_2}W,$$

$$\Pi(3,7)=6e^{-2\alpha h_2}W,$$

$$\Pi(4,4)=-e^{-2\alpha h_2}Z-e^{2\alpha h_2}W_2-4e^{-2\alpha h_2}W,$$

$$\Pi(4,6)=6e^{-2\alpha h_2}W,$$

$$\Pi(5,5)=-(h_1^2W_1+h_2^2W_2+h_{12}^2W),$$

$$\Pi(5,8) = (h_1^2 W_1 + h_2^2 W_2 + h_{12}^2 W)H,$$
$$\Pi(6,6) = -12e^{-2\alpha h_2} W,$$
$$\Pi(7,7) = -12e^{-2\alpha h_2} W,$$
$$\Pi(8,8) = -\varepsilon I.$$

Theorem 1. *For given scalars* $\beta > 0, 0 < h_1 \le h_2$ *and* $\alpha > 0$.

System (1) is $\alpha -$*exponentially if there exist symmetric positive definite matrices* P, Q, Z, W, W_i $(i = 1,2)$ *and any number* $\varepsilon > 0$ *satisfying the following LMI:*

$$\begin{bmatrix} \Pi(1,1) & \Pi(1,2) & 0 & 0 & \Pi(1,5) & 0 & 0 & \Pi(1,8) \\ \star & \Pi(2,2) & \Pi(2,3) & \Pi(2,4) & \Pi(2,5) & \Pi(2,6) & \Pi(2,7) & 0 \\ \star & \star & \Pi(3,3) & 0 & 0 & 0 & \Pi(3,7) & 0 \\ \star & \star & \star & \Pi(4,4) & 0 & \Pi(4,6) & 0 & 0 \\ \star & \star & \star & \star & \Pi(5,5) & 0 & 0 & \Pi(5,8) \\ \star & \star & \star & \star & \star & \Pi(6,6) & 0 & 0 \\ \star & \star & \star & \star & \star & \star & \Pi(7,7) & 0 \\ \star & \star & \star & \star & \star & \star & \star & \Pi(8,8) \end{bmatrix} < 0. \quad (4)$$

Proof. We construct the following Lyapunov-Krasovskii function (LKF)

$$V(t, x_t) = V_1 + V_2 + V_3 + V_4 + V_5 + V_6 \quad (5)$$

where
$$V_1 = x^{\mathrm{T}}(t)Px(t),$$
$$V_2 = \int_{t-h(t)}^{t} e^{2\alpha(s-t)} x^{\mathrm{T}}(s)Qx(s)ds,$$
$$V_3 = \int_{t-h_2}^{t-h_1} e^{2\alpha(s-t)} x^{\mathrm{T}}(s)Zx(s)ds,$$
$$V_4 = h_1 \int_{-h_1}^{0} \int_{t+s}^{t} e^{2\alpha(u-t)} \dot{x}^{\mathrm{T}}(u)W_1\dot{x}(u)duds,$$
$$V_5 = h_2 \int_{-h_2}^{0} \int_{t+s}^{t} e^{2\alpha(u-t)} \dot{x}^{\mathrm{T}}(u)W_2\dot{x}(u)duds,$$
$$V_6 = h_{12} \int_{-h_2}^{-h_1} \int_{t+s}^{t} e^{2\alpha(u-t)} \dot{x}^{\mathrm{T}}(u)W\dot{x}(u)duds.$$

It is easy to see that $\lambda_1 \| x(t) \|^2 \le V(t, x_t) \le \lambda_2 \| x_t \|^2,$ (6)

where x_t denotes the segment $\{x(t+s) : s \in [\tau; 0]\}, \lambda_1 = \lambda_{min}(P)$ and

$$\lambda_2 = \lambda_{max}(P) + e^{-2\alpha h_1}\lambda_{max}(Z) + h_2\lambda_{max}(Q) + \frac{h_1^3}{2}\lambda_{max}(W_1) + \frac{h_2^3}{2}\lambda_{max}(W_2) + \frac{h_{12}^2(h_1 + h_2)}{2}\lambda_{max}(W).$$

Taking derivative of V_1 in t along the trajectory of the system, we have

$$\dot{V}_1 = 2x^T(t)P\dot{x}(t) = 2x^{\mathrm{T}}(t)P[Ax(t) + Bx(t-h(t)) + Hg(t, x(t), x(t-h(t)))] \tag{7}$$
$$= x^{\mathrm{T}}(t)[PA + A^{\mathrm{T}}P^{\mathrm{T}} + 2\alpha P]x(t) + 2x^{\mathrm{T}}(t)PBx(t-h(t)) + 2x^{\mathrm{T}}(t)PHg(t, x(t), x(t-h(t))) - 2\alpha V_1.$$

From (2), it is easy to see that

$$\varepsilon(a^2 x(t)^T E^T Ex(t) + b^2 x(t-h(t))^T F^T Fx(t-h(t)) - g^T(.)g(.) \ge 0 \tag{8}$$

where any $\varepsilon > 0$. From (7) and (8), we have

$$\dot{V} \le x^{\mathrm{T}}(t)[PA + A^{\mathrm{T}}P^{\mathrm{T}} + 2\alpha P]x(t) + 2x^{\mathrm{T}}(t)PBx(t-h(t)) + 2x^{\mathrm{T}}(t)PHg(t, x(t), x(t-h(t))) - 2\alpha V_1 \tag{9}$$
$$+\varepsilon(a^2 x(t)^{\mathrm{T}} E^{\mathrm{T}} Ex(t) + b^2 x(t-h(t))^{\mathrm{T}} F^{\mathrm{T}} Fx(t-h(t)) - g^{\mathrm{T}}(.)g(.)).$$

Next, the time-derivative of $V_k, k = 2,3,...,8,$ are computed and estimated as follows

$$\dot{V}_2 = x^{\mathrm{T}}(t)Qx(t) - (1-\dot{h}(t))e^{-2\alpha h(t)}x^{\mathrm{T}}(t-h(t))Qx(t-h(t)) - 2\alpha V_2; \tag{10}$$
$$\le x^{\mathrm{T}}(t)Qx(t) - (1-\beta)e^{-2\alpha h_2}x^{\mathrm{T}}(t-h(t))Qx(t-h(t)) - 2\alpha V_2;$$

$$\dot{V}_3 = e^{-2\alpha h_1}x^{\mathrm{T}}(t-h_1)Zx(t-h_1) - e^{-2\alpha h_2}x^{\mathrm{T}}(t-h_2)Zx(t-h_2) - 2\alpha V_3; \tag{11}$$

$$\dot{V}_4 = h_1^2\dot{x}^{\mathrm{T}}(t)W_1\dot{x}(t) - h_1\int_{t-h_1}^{t} e^{2\alpha(s-t)}\dot{x}^{\mathrm{T}}(s)W_1\dot{x}(s)ds - 2\alpha V_4 \tag{12}$$
$$\le h_1^2\dot{x}^{\mathrm{T}}(t)W_1\dot{x}(t) - h_1 e^{-2\alpha h_1}\int_{t-h_1}^{t}\dot{x}^{\mathrm{T}}(s)W_1\dot{x}(s)ds - 2\alpha V_4;$$

$$\dot{V}_5 = h_2^2\dot{x}^{\mathrm{T}}(t)W_2\dot{x}(t) - h_2\int_{t-h_2}^{t} e^{2\alpha(s-t)}\dot{x}^{\mathrm{T}}(s)W_2\dot{x}(s)ds - 2\alpha V_5 \tag{13}$$
$$\le h_2^2\dot{x}^{\mathrm{T}}(t)W_2\dot{x}(t) - h_2 e^{-2\alpha h_2}\int_{t-h_2}^{t}\dot{x}^{\mathrm{T}}(s)W_2\dot{x}(s)ds - 2\alpha V_5;$$

$$\dot{V}_6 = h_{12}^2\dot{x}^{\mathrm{T}}(t)W\dot{x}(t) - h_{12}\int_{t-h_2}^{t-h_1} e^{2\alpha(s-t)}\dot{x}^{\mathrm{T}}(s)W\dot{x}(s)ds - 2\alpha V_6 \tag{14}$$
$$\le h_{12}^2\dot{x}^{\mathrm{T}}(t)W\dot{x}(t) - h_{12} e^{-2\alpha h_2}\int_{t-h_2}^{t-h_1}\dot{x}^{\mathrm{T}}(s)W\dot{x}(s)ds - 2\alpha V_6.$$

Applying the Proposition 2, we have

$$-h_1\int_{t-h_1}^{t}\dot{x}^{\mathrm{T}}(s)W_1\dot{x}(s)ds \le -[x(t) - x(t-h_1)]^{\mathrm{T}}W_1[x(t) - x(t-h_1)] \tag{15}$$

and $$-h_2\int_{t-h_2}^{t}\dot{x}^{\mathrm{T}}(s)W_2\dot{x}(s)ds \le -[x(t) - x(t-h_2)]^{\mathrm{T}}W_2[x(t) - x(t-h_2)] \tag{16}$$

Besides, we have

$$-h_{12}\int_{t-h_2}^{t-h_1}\dot{x}^{\mathrm{T}}(s)W\dot{x}(s)ds = -h_{12}\int_{t-h_2}^{t-h(t)}\dot{x}^{\mathrm{T}}(s)W\dot{x}(s)ds - h_{12}\int_{t-h(t)}^{t-h_1}\dot{x}^{\mathrm{T}}(s)W\dot{x}(s)ds.$$

Applying the Proposition 2, we have

$$
\begin{aligned}
&-h_{12}\int_{t-h_2}^{t-h(t)}\dot{x}^{\mathrm{T}}(s)W\dot{x}(s)ds \qquad (17)\\
&\le -\frac{h_{12}}{h_2-h(t)}[(x(t-h(t))-x(t-h_2))^{\mathrm{T}}W(x(t-h(t))-x(t-h_2))\\
&+3(x(t-h(t))+x(t-h_2))^{\mathrm{T}}W(x(t-h(t))+x(t-h_2))\\
&+12(\frac{1}{h_2-h(t)}\int_{t-h_2}^{t-h(t)}x(s)ds)^{\mathrm{T}}W(\frac{1}{h_2-h(t)}\int_{t-h_2}^{t-h(t)}x(s)ds)\\
&-12(x(t-h(t))+x(t-h_2))^{\mathrm{T}}W_2(\frac{1}{h_2-h(t)}\int_{t-h_2}^{t-h(t)}x(s)ds)]\\
&\le -4x^{\mathrm{T}}(t-h(t))Wx(t-h(t))\\
&-4x^{\mathrm{T}}(t-h_2)Wx(t-h_2)-4x^{\mathrm{T}}(t-h(t))W_2x(t-h_2)\\
&+12x^{\mathrm{T}}(t-h(t))W(\frac{1}{h_2-h(t)}\int_{t-h_2}^{t-h(t)}x(s)ds)\\
&+12x^{\mathrm{T}}(t-h_2)W(\frac{1}{h_2-h(t)}\int_{t-h_2}^{t-h(t)}x(s)ds)\\
&-12(\frac{1}{h_2-h(t)}\int_{t-h_2}^{t-h(t)}x(s)ds)^{\mathrm{T}}W(\frac{1}{h_2-h(t)}\int_{t-h_2}^{t-h(t)}x(s)ds).
\end{aligned}
$$

Similarly, we have

$$
\begin{aligned}
&-h_{12}\int_{t-h(t)}^{t-h_1}\dot{x}^{\mathrm{T}}(s)W\dot{x}(s)ds \qquad (18)\\
&\le -\frac{h_{12}}{h(t)-h_1}[(x(t-h_1)-x(t-h(t)))^{\mathrm{T}}W(x(t-h_1)-x(t-h(t)))\\
&+3(x(t-h_1)+x(t-h(t)))^{\mathrm{T}}W(x(t-h_1)+x(t-h(t)))\\
&+12(\frac{1}{h(t)-h_1}\int_{t-h(t)}^{t-h_1}x(s)ds)^{\mathrm{T}}W(\frac{1}{h(t)-h_1}\int_{t-h(t)}^{t-h_1}x(s)ds)\\
&-12(x(t-h_1)+x(t-h(t)))^{\mathrm{T}}W(\frac{1}{h(t)-h_1}\int_{t-h(t)}^{t-h_1}x(s)ds)]\\
&\le -4x^{\mathrm{T}}(t-h_1)Wx(t-h_1)-4x^{\mathrm{T}}(t-h(t))Wx(t-h(t))\\
&-4x^{\mathrm{T}}(t-h_1)Wx(t-h(t))\\
&+12x^{\mathrm{T}}(t-h_1)W(\frac{1}{h(t)-h_1}\int_{t-h(t)}^{t-h_1}x(s)ds)\\
&+12x^{\mathrm{T}}(t-h(t))W(\frac{1}{h(t)-h_1}\int_{t-h(t)}^{t-h_1}x(s)ds)\\
&-12(\frac{1}{h(t)-h_1}\int_{t-h(t)}^{t-h_1}x(s)ds)^{\mathrm{T}}W(\frac{1}{h(t)-h_1}\int_{t-h(t)}^{t-h_1}x(s)ds).
\end{aligned}
$$

On the other hand, using the following identities

$-\dot{x}(t)+Ax(t)+Bx(t-h(t))+Hg(t,x(t),x(t-h(t)))=0,$ we obtain

$$2\dot{x}(t)^{T}[h_1^2W_1+h_2^2W_2+h_{12}^2W][-\dot{x}(t)+Ax(t)+Bx(t-h(t))+Hg(t,x(t),x(t-h(t)))]=0. \quad (19)$$

Therefore, from (9) to (21), it implies $\dot{V}(t,x_t)+2\alpha V(t,x_t)\le \Upsilon^{T}(t)\Pi\Upsilon(t), \quad \forall t\ge 0,$ (20)

where $\Pi=\begin{bmatrix} \Pi(1,1) & \Pi(1,2) & 0 & 0 & \Pi(1,5) & 0 & 0 & \Pi(1,8)\\ \star & \Pi(2,2) & \Pi(2,3) & \Pi(2,4) & \Pi(2,5) & \Pi(2,6) & \Pi(2,7) & 0\\ \star & \star & \Pi(3,3) & 0 & 0 & 0 & \Pi(3,7) & 0\\ \star & \star & \star & \Pi(4,4) & 0 & \Pi(4,6) & 0 & 0\\ \star & \star & \star & \star & \Pi(5,5) & 0 & 0 & \Pi(5,8)\\ \star & \star & \star & \star & \star & \Pi(6,6) & 0 & 0\\ \star & \star & \star & \star & \star & \star & \Pi(7,7) & 0\\ \star & \star & \star & \star & \star & \star & \star & \Pi(8,8) \end{bmatrix}$

from inequality (4), we have $\Pi<0$. Consequently $\dot{V}(t,x_t)+2\alpha V(t,x_t)\le 0, \forall t\ge 0,$ and then $V(t,x_t)\le V(0,\phi)e^{-2\alpha t}\le \lambda_2\|\phi\|_{\tau}^2 e^{-2\alpha t}, \quad t\ge 0.$ Taking (6) into account, we obtain

$$\|x(t)\|\le\sqrt{\frac{\lambda_2}{\lambda_1}}\|\phi\|_{\tau}e^{-\alpha t}:=N\|\phi\|_{\tau}e^{-\alpha t}, \quad t\ge 0. \quad (21)$$

Remark 1. *If β is unknown or $h(t)$ is not differentiable, then the following result can be obtained from Theorem 1 by setting $Q=0$, which will be introduced as Corollary 1.*

Corollary 1. *For given scalars $0<h_1\le h_2$ and $\alpha>0$. System (1) is $\alpha-$ exponentially if there exist symmetric positive definite matrices P,Z,W,W_i $(i=1,2)$ and any number $\varepsilon>0$ satisfying the following LMI:*

$$\begin{bmatrix} \Pi(1,1) & \Pi(1,2) & 0 & 0 & \Pi(1,5) & 0 & 0 & \Pi(1,8)\\ \star & \Pi(2,2) & \Pi(2,3) & \Pi(2,4) & \Pi(2,5) & \Pi(2,6) & \Pi(2,7) & 0\\ \star & \star & \Pi(3,3) & 0 & 0 & 0 & \Pi(3,7) & 0\\ \star & \star & \star & \Pi(4,4) & 0 & \Pi(4,6) & 0 & 0\\ \star & \star & \star & \star & \Pi(5,5) & 0 & 0 & \Pi(5,8)\\ \star & \star & \star & \star & \star & \Pi(6,6) & 0 & 0\\ \star & \star & \star & \star & \star & \star & \Pi(7,7) & 0\\ \star & \star & \star & \star & \star & \star & \star & \Pi(8,8) \end{bmatrix}<0. \quad (22)$$

where $\Pi(1,1)=PA+A^{T}P^{T}+2\alpha P+\varepsilon a^2E^{T}E-e^{-2\alpha h_1}W_1-e^{-2\alpha h_2}W_2,$

$$\Pi(2,2)=-8e^{-2\alpha h_2}W+\varepsilon b^2F^{T}F,$$

In other cases, $\Pi(i,j)$ are defined as in Theorem 1.

4. Conclusion

This paper has dealt with the problem of exponential stability analysis for a class of nonlinear systems with interval time-varying delays. A constructive approach and new delay-dependent condition in terms of linear matrix inequality have been proposed based on an improved LKF. Our condition guarantees the exponential stability of the system with special exponential delay rate.

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DETERMINED ABSORPTION COEFFICIENT OF ^{85}Rb ATOM IN THE Y-CONFIGURATION

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Received: 12 May 2019/ Accepted: 11 June 2019/ Published: June 2019

Abstract: *In this work, we establish a system of equations of density and derive analytical expression for the absorption coefficient of ^{85}Rb atomin the Y -configuration for a weak probe laser beam induced by two strong coupling laser beams. Our results show possible ways to control absorption coefficient by frequency detuning probe laser and intensity of the coupling laser.*

Keywords: *Electromagnetically induced transparency, absorption coefficient.*

1. Introduction

The manipulation of subluminal and superluminal light propagation in optical medium has attracted many attentions due to its potential applications during the last decades, such as controllable optical delay lines, optical switching [2], telecommunication, interferometry, optical data storage and optical memories quantum information processing, and so on [6]. The most important key to manipulate subluminal and superluminal light propagations lies in its ability to control the absorption and dispersion properties of a medium by a laser field.

As we know that coherent interaction between atom and light field can lead to interesting quantum interference effects such as electromagnetically induced transparency (EIT) [1]. The EIT is a quantum interference effect between the probability amplitudes that leads to a reduction of resonant absorption for a weak probe light field propagating through a medium induced by a strong coupling light field [5]. Basic configurations of the EIT effect are three-level atomic systems including the Λ-Ladder and V-type configurations. In each configuration, the EIT efficiency is different, in which the Λ-type configuration is the best, whereas the V-type configuration is the worst [4], [7], therefore, the manipulation of light in each configuration are also different. This suggests that we choose to use the analytical model to determine the absorption coefficient for the Y configuration of the ^{85}Rb atomic system.

2. The density matrix equation for ^{85}Rb atomic system configure Y

We first consider a Y-configuration of ^{85}Rb atom as shown in Fig. 1. State $|1\rangle$ is the ground states of the level $5S_{1/2}$ (F=3). The $|2\rangle$, $|3\rangle$ and $|4\rangle$ states are excited states of the levels $5P_{3/2}$ (F'=3), $5D_{5/2}$ (F"=4) and $5D_{5/2}$ (F"=3) [7].

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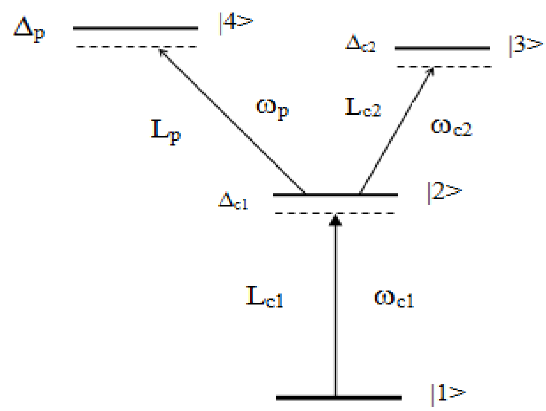

Figure 1. *Four-level excitation of the Y- configuration.*

Put this Y-configuration into three laser beams atomic frequency and intensity appropriate: a week probe laser L_p has intensity E_p with frequency ω_p applies the transition $|2\rangle \;\forall\; |4\rangle$ and the Rabi frequencies of the probe $\Omega_p = \frac{\mu_{42} E_p}{\hbar}$; Two strong coupling laser L_{c1} and L_{c2} couple the transition $|1\rangle \;\forall |2\rangle$ and $|2\rangle \forall |3\rangle$ the Rabi frequencies of the two coupling fields $\Omega_{c1} = \frac{\mu_{21} E_{c1}}{\hbar}$ and $\Omega_{c2} = \frac{\mu_{32} E_{c2}}{\hbar}$, where μ_{ij} is the electric dipole matrix element $|i\rangle \leftrightarrow |j\rangle$. The evolution of the system, which is represented via the density operator ρ is determined by the following Liouville equation [2]:

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar}[H, \rho] + \Lambda\rho, \tag{1}$$

where, H represents the total Hamiltonian and $\Lambda\rho$ represents the decay part. Hamilton of the systerm can be written by matrix form:

$$H = \begin{pmatrix} \hbar\omega_1 & \frac{\hbar\Omega_{c1}}{2} e^{i\omega_{c1}t} & 0 & 0 \\ \frac{\hbar\Omega_{c1}}{2} e^{-i\omega_{c1}t} & \hbar\omega_2 & \frac{\hbar\Omega_{c2}}{2} e^{i\omega_{c2}t} & \frac{\hbar\Omega_p}{2} e^{i\omega_p t} \\ 0 & \frac{\hbar\Omega_{c2}}{2} e^{-i\omega_{c2}t} & \hbar\omega_3 & 0 \\ 0 & \frac{\hbar\Omega_p}{2} e^{-i\omega_p t} & 0 & \hbar\omega_4 \end{pmatrix} \tag{2}$$

We consider the slow variation and put: $\rho_{43} = \tilde{\rho}_{43} e^{-i(\omega_p - \omega_{c2})t}$, $\rho_{42} = \tilde{\rho}_{42} e^{-i\omega_p t}$, $\rho_{41} = \tilde{\rho}_{41} e^{-i(\omega_p + \omega_{c1})t}$, $\rho_{32} = \tilde{\rho}_{32} e^{-i\omega_{c2}t}$, $\rho_{31} = \tilde{\rho}_{31} e^{-i(\omega_{c1} + \omega_{c2})t}$, $\rho_{21} = \tilde{\rho}_{21} e^{-i\omega_{c1}t}$. In the framework of the semiclassical theory, the density matrix equations can be written as:

$$\dot{\rho}_{44} = \frac{i\Omega_p}{2}\left(\tilde{\rho}_{42} - \tilde{\rho}_{24}\right) - \Gamma_{43}\rho_{44} \tag{3.1}$$

$$\dot{\tilde{\rho}}_{41} = \frac{i\Omega_{c1}}{2}\tilde{\rho}_{42} - \frac{i\Omega_p}{2}\rho_{21} + [i(\Delta_{c1} + \Delta_p) - \gamma_{41}]\tilde{\rho}_{41} \tag{3.2}$$

$$\dot{\tilde{\rho}}_{42} = \frac{i\Omega_{c1}}{2}\tilde{\rho}_{41} + \frac{i\Omega_{c2}}{2}\tilde{\rho}_{43} + \frac{i\Omega_p}{2}(\rho_{44} - \rho_{22}) + (i\Delta_p - \gamma_{42})\tilde{\rho}_{42} \tag{3.3}$$

$$\dot{\tilde{\rho}}_{43} = \frac{i\Omega_{c2}}{2}\tilde{\rho}_{42} - \frac{i\Omega_p}{2}\tilde{\rho}_{23} + [i(\Delta_p - \Delta_{c2}) - \gamma_{43}]\tilde{\rho}_{43} \tag{3.4}$$

$$\dot{\rho}_{33} = \frac{i\Omega_{c2}}{2}\left(\tilde{\rho}_{32} - \tilde{\rho}_{23}\right) + \Gamma_{43}\rho_{44} - \Gamma_{32}\rho_{33} \tag{3.5}$$

$$\dot{\tilde{\rho}}_{31} = \frac{i\Omega_{c1}}{2}\tilde{\rho}_{32} - \frac{i\Omega_{c2}}{2}\tilde{\rho}_{21} + [i(\Delta_p + \Delta_{c1}) - \gamma_{31}]\tilde{\rho}_{31} \tag{3.6}$$

$$\dot{\tilde{\rho}}_{32} = \frac{i\Omega_{c1}}{2}\tilde{\rho}_{31} + \frac{i\Omega_{c2}}{2}\left(\rho_{33} - \rho_{22}\right) + \frac{i\Omega_p}{2}\tilde{\rho}_{34} + (i\Delta_{c2} - \gamma_{32})\tilde{\rho}_{32} \tag{3.7}$$

$$\dot{\tilde{\rho}}_{34} = \frac{i\Omega_p}{2}\tilde{\rho}_{32} - \frac{i\Omega_{c2}}{2}\tilde{\rho}_{24} + [-i(\Delta_p - \Delta_{c2}) - \gamma_{43}]\tilde{\rho}_{34} \tag{3.8}$$

$$\dot{\rho}_{22} = \frac{i\Omega_{c1}}{2}\left(\tilde{\rho}_{21} - \tilde{\rho}_{12}\right) + \frac{i\Omega_{c2}}{2}\left(\tilde{\rho}_{23} - \tilde{\rho}_{32}\right) + \frac{i\Omega_p}{2}\left(\tilde{\rho}_{24} - \tilde{\rho}_{42}\right) + \Gamma_{32}\rho_{33} - \Gamma_{21}\rho_{22} \tag{3.9}$$

$$\dot{\tilde{\rho}}_{21} = \frac{i\Omega_{c1}}{2}\left(\rho_{22} - \rho_{11}\right) - \frac{i\Omega_{c2}}{2}\tilde{\rho}_{31} - \frac{i\Omega_p}{2}\tilde{\rho}_{41} + (i\Delta_{c1} - \gamma_{21})\tilde{\rho}_{21} \tag{3.10}$$

$$\dot{\tilde{\rho}}_{23} = \frac{i\Omega_{c2}}{2}\left(\rho_{22} - \rho_{33}\right) - \frac{i\Omega_{c1}}{2}\tilde{\rho}_{13} - \frac{i\Omega_p}{2}\tilde{\rho}_{43} + (-i\Delta_{c2} - \gamma_{32})\tilde{\rho}_{23} \tag{3.11}$$

$$\dot{\tilde{\rho}}_{24} = \frac{i\Omega_p}{2}(\rho_{22} - \rho_{44}) - \frac{i\Omega_{c1}}{2}\tilde{\rho}_{14} - \frac{i\Omega_{c2}}{2}\tilde{\rho}_{34} + (-i\Delta_p - \gamma_{42})\tilde{\rho}_{24} \tag{3.12}$$

$$\dot{\rho}_{11} = \frac{i\Omega_{c1}}{2}\left(\tilde{\rho}_{12} - \tilde{\rho}_{21}\right) + \Gamma_{21}\rho_{22} \tag{3.13}$$

$$\dot{\tilde{\rho}}_{12} = \frac{i\Omega_{c1}}{2}\left(\rho_{11} - \rho_{22}\right) + \frac{i\Omega_{c2}}{2}\tilde{\rho}_{13} + \frac{i\Omega_p}{2}\tilde{\rho}_{14} + (i\Delta_{c1} - \gamma_{21})\tilde{\rho}_{12} \tag{3.14}$$

$$\dot{\tilde{\rho}}_{13} = \frac{i\Omega_{c2}}{2}\tilde{\rho}_{12} - \frac{i\Omega_{c1}}{2}\tilde{\rho}_{23} + [-i(\Delta_{c1} + \Delta_{c2}) - \gamma_{31}]\tilde{\rho}_{13} \tag{3.15}$$

$$\dot{\tilde{\rho}}_{14} = \frac{i\Omega_p}{2}\tilde{\rho}_{12} - \frac{i\Omega_{c1}}{2}\tilde{\rho}_{24} + [-i(\Delta_p + \Delta_{c1}) - \gamma_{41}]\tilde{\rho}_{14} \tag{3.16}$$

(where, the frequency detuning of the probe and L_{c1}, L_{c2} coupling lasers from the relevant atomic transitions are respectively determined by $\Delta_p = \omega_p - \omega_{42}$, $\Delta_{c1} = \omega_{c1} - \omega_{21}$. In addition, suppose the initial atomic system is at a level $|2\rangle$ therefore: $\rho_{11} \approx \rho_{33} \approx \rho_{44} \approx 0, \rho_{22} = 1$.

Now, we analytically solve the density matrix equations under the steady-state condition by setting the time derivatives to zero:

$$\frac{d\rho}{dt} = 0, \tag{4}$$

Therefore the equations (3.2), (3.3) and (3.4), we have:

$$0 = \frac{i\Omega_{c1}}{2}\tilde{\rho}_{42} - \frac{i\Omega_p}{2}\rho_{21} + [i(\Delta_{c1} + \Delta_p) - \gamma_{41}]\tilde{\rho}_{41} \tag{5.1}$$

$$0 = \frac{i\Omega_{c1}}{2}\tilde{\rho}_{41} + \frac{i\Omega_{c2}}{2}\tilde{\rho}_{43} + \frac{i\Omega_p}{2}(\rho_{44} - \rho_{22}) + (i\Delta_p - \gamma_{42})\tilde{\rho}_{42} \tag{5.2}$$

$$0 = \frac{i\Omega_{c2}}{2}\tilde{\rho}_{42} - \frac{i\Omega_p}{2}\tilde{\rho}_{23} + [i(\Delta_p - \Delta_{c2}) - \gamma_{43}]\tilde{\rho}_{43} \tag{5.3}$$

Because of $\Omega_p << \Omega_{c1}$ and Ω_{c2} so that we ignore the term $\frac{i\Omega_p}{2}\tilde{\rho}_{21}$ and $\frac{i\Omega_p}{2}\tilde{\rho}_{23}$ in the equations (4) and (5). Slove the equations (4) – (5), we have:

$$\tilde{\rho}_{42} = \frac{-i\Omega_p/2}{\gamma_{42} - i\Delta_p + \frac{\Omega_{c1}^2/4}{\gamma_{41} - i(\Delta_p + \Delta_{c1})} + \frac{\Omega_{c2}^2/4}{\gamma_{43} - i(\Delta_p - \Delta_{c2})}}, \tag{6}$$

3. Absorption coefficient of the atomic medium

We start from the susceptibility of atomic medium for the probe light that is determined by the following relation:

$$\chi = -2\frac{Nd_{21}}{\varepsilon_0 E_p}\rho_{21} = \chi' + i\chi'', \tag{7}$$

The absorption coefficient α of the atomic medium for the probe beam is determined through the imaginary part of the linear susceptibility (7):

$$\alpha = \frac{\chi''\omega_p}{c} = -\frac{\omega_p}{c}\frac{2N\mu_{42}^2}{\Omega_p\hbar\varepsilon_0}\mathrm{Im}(\tilde{\rho}_{42}) \tag{8}$$

We considere the case of ^{85}Rb atomic: γ_{42} = 3MHz, γ_{41} = 0.3MHz and γ_{43} = 0.03MHz, the atomic density $N = 10^{11}/\mathrm{cm}^3$. The electric dipole matrix element is $d_{42} = 2.54.10^{-29}$ Cm, dielectric coefficient $\varepsilon_0 = 8.85.10^{-12}$ F/m, $\hbar = 1{,}05.10^{-34}$ J.s, and frequency of probe beam $\omega_p = 3.84.10^{14}$ Hz. Fixed frequency Rabi of coupling laser beam L_{c1} in value Ω_{c1} = 16MHz (correspond to the value that when there is no laser L_{c2} then the transparency of the probe beam near 100%) and the frequency coincides with the frequency of the transition $|1\rangle \leftrightarrow |2\rangle$, it means $\Delta_{c1} = 0$. Consider the case of the frequency deviation of the coupling laser beam L_{c2} is Δ_{c2} = 10MHz . We plot a three-dimensional graph of the absorption coefficient α at the intensity of the coupling laser beam L_{c2} (Rabi frequency Ω_{c2}) and and the frequency deviation of the probe laser beam L_p, the result is shown in Fig 2.

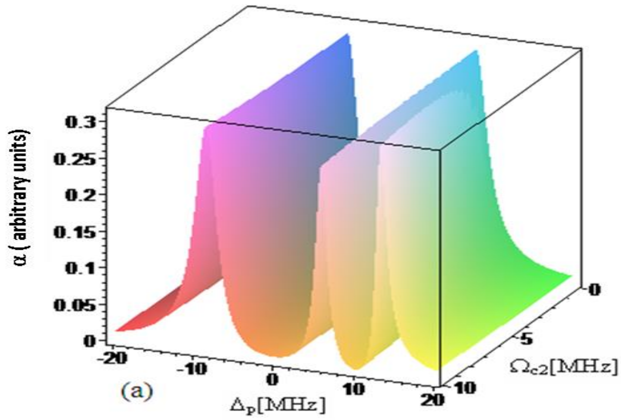

Figure 2. *Three-dimensional graph of the absorption coefficient α according to Δp and Ωc2 with Δc1 = 0 MHz*

As shown in Fig 2, we see that when there is no coupling laser beam, it makes L_{c2} (Ω_{c2} = 0) then our model is only a three-step configuration [5], [6], we have only one transparent window at the resonant frequency of the probe laser beam. When presenting in the coupling laser beam L_{c2} (with the frequency deviation chosen is Δ_{c2} = 10MHz) and gradually increasing Rabi frequency Ω_{c2}, we see a window appear more during time the absorber envelopes at frequency deviation of probe beam Δ_p = 10MHz (satisfy the condition of two-photon resonance with the laser beam L_p and L_{c2} is $\Delta_p - \Delta_{c2} = 0$), and the depth and width of this transparent window also increases with the increase of Ω_{c2}.

To be more specific, we plot a two-dimensional graph of Figure 3 with some specific values of Rabi frequency Ω_{c2}.

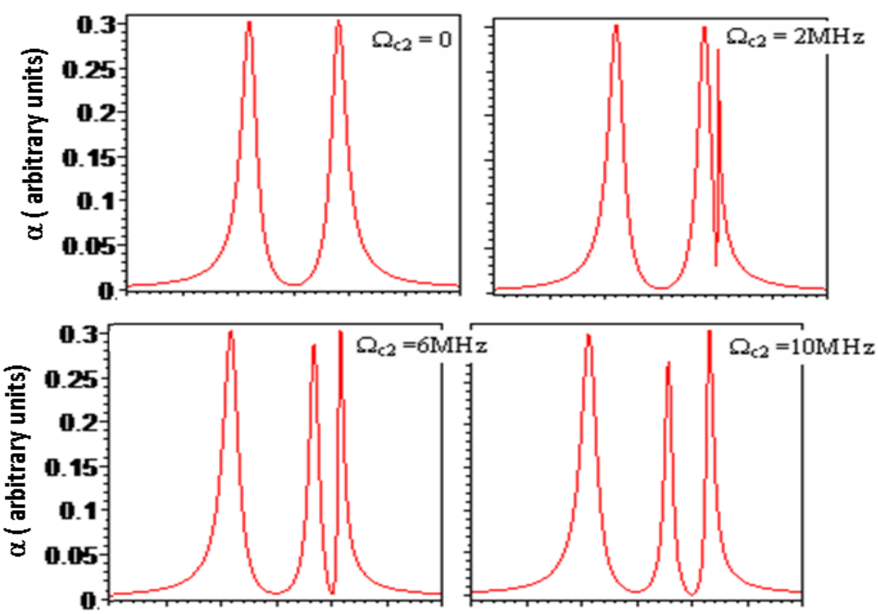

Figure 3. *Two-dimensional graph* of *the absorption coefficient α according to* Ω_{c2} with Ω_{c1} = 16MHz , Δ_{c1} = 0 and Δ_{c2} = 10MHz.

4. Conclusion

In the framework of the semi-classical theory, we have cited the density matrix equation for the ^{85}Rb atomic system in the Y-configuration under the simultaneous effects of two laser probe and coupling beams. Using approximate rotational waves and approximate electric dipoles, we have found solutions in the form of analytic for the absorption coefficient of atoms when the probe beam has a small intensity compared to the coupling beams. Drawing the absorption coefficient expression will facilitate future research applications. Consequently, we investigated the absorption of the probe beam according to the intensity of the coupling beam Ω_{c1}, Ω_{c2} and the deviation of the probe beam Δ_p. The results show that a Y-configuration appears two transparent windows for the probe laser beam. The depth and width or position of these windows can be altered by changing the intensity or frequency deviations of the coupling laser fields.

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IDENTICAL SYNCHRONIZATION IN COMPLETE NETWORK OF ORDINARY DIFFERENTIAL EQUATIONS OF FITZHUGH-NAGUMO

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Received: 10 January 2019/ Accepted: 11 June 2019/ Published: June 2019

Abstract: *Synchronization is a ubiquitous feature in many natural systems and nonlinear science. In this paper, the synchronization in complete network consisting of n nodes is studied. Each node is connected to all other nodes by linear coupling and it is represented by a system of ordinary differential equations of FitzHugh-Nagumo type which is obtained by simplifying the famous Hodgkin-Huxley model. From this complete network, the sufficient condition under the coupling strength is sought such that the synchronization phenomenon occurs. The result shows that the networks with bigger in-degrees of the nodes synchronize more easily. The paper also shows this theoretical result numerically and see that there is a compromise.*

Keywords: *Coupling strength, complete network, FitzHugh-Nagumo model, synchronization.*

1. Introduction

The FitzHugh-Nagumo model was introduced as a dimensional reduction of the well-known Hodgkin-Huxley model [6], [7], [9], [10], [11], [12]. It is more analytically tractable and it maintains a certain biophysical meaning. The model is constituted by two equations in two variables u and v. The first one is the fast variable called excitatory: it represents the transmembrane voltage. The second variable is the slow recovery variable: it describes the time dependence of several physical quantities, such as the electrical conductance of the ion currents across the membrane. The FitzHugh-Nagumo equations (FHN), using the notation in [1], [2], [3], are given by

$$\begin{cases} \varepsilon \dfrac{du}{dt} = u_t = f(u) - v \\ \dfrac{dv}{dt} = v_t = au - bv + c \end{cases} \qquad (1)$$

where a, b and c are constants (a and b are strictly positive), $0 < \varepsilon \ll 1$ and $f(u) = -u^3 + 3u$, $t \in \mathbb{R}^+$ presents the time. The system (1) is the model of a neuron, then we consider a network of n coupled systems (1) based on FHN type as follows:

$$\begin{cases} \varepsilon u_{it} = \varepsilon \dfrac{du_i}{dt} = f(u_i) - v_i - h(u_i, v_i) \\ v_{it} = \dfrac{dv_i}{dt} = au_i - bv_i + c \end{cases} \quad i, j = 1, \ldots, n, i \neq j, \qquad (2)$$

where $(u_i, v_i), i = 1, 2, \ldots, n$ is defined by (1).

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The function *h* is the coupling function that determines the type of connection between neurons *i* and *j*. Connections between neurons are essentially of two types: chemical which is much more abundant, and electrical. In the case where the connections are made by electrical synapse, the coupling is linear and given by the function

$$h(u_i, v_i) = g_{syn} \sum_{j=1}^{n} c_{ij}(u_i - u_j), \quad i = 1, 2, ..., n. \tag{3}$$

The parameter g_{syn} represents the coupling strength. The coefficients c_{ij} are the elements of the connectivity matrix $C_n = (c_{ij})_{n\times n}$, defined by

$$\begin{cases} c_{ij} = 1 \text{ if } i \text{ and } j \text{ are coupled} \\ c_{ij} = 0 \text{ if } i \text{ and } j \text{ are not coupled} \end{cases} \qquad i, j = 1, 2, ..., n, i \neq j.$$

A neural network describes a population of physically interconnected nerve cells. Communication between cells is mainly due to electrochemical processes. This work focuses on analyzing the behavior of a set of neurons connected together with a given topology by electrical. Thus, the complex system based on a network of interactions between neurons is considered in which each network node is modeled by a ODE of FHN type.

The article is divided as follows: in section 1, there is the introduction; in section 2, the definition of synchronization is introduced, especially identical synchronization. And this paper looks forward to finding out the sufficient conditions such that there is a such type of synchronization in network; in section 3, this research focuses on the minimal value of coupling strength such that the synchronization in complete network occurs and numerical experiments to give an insight into the influence of the number of neurons on the minimal coupling strength needed to obtain synchronization in network. The numerical simulations show that when the number of nodes in graph grows, the network becomes easier to synchronize. And the conclusion is left to the last section.

2. Identical synchronization of a complete network of *n* systems of ordinary differential equations on FitzHugh-Nagumo type

Synchronization is a ubiquitous feature in many natural systems and nonlinear science. The word "*synchronization*" is from the Greek, *syn* (common) and *chronos* (time), and means having the same behavior at the same time. Therefore the synchronization of two dynamical systems usually means that one system copies the movement of the other. When the behavior of many systems is synchronized, these systems are called *synchronous*. It is known that a phenomenon of synchronization may appear in a network of many weak coupled oscillators [4], [5], [13]. A broad variety of applications have emerged, for example to increase the power of lasers, to synchronize the output of electric circuits, to control oscillations in chemical reactions or to encode electronic messages for secure communications. Here are some synchronization regimes:

Identical (or complete) synchronization, which is defined as the coincidence of states of interacting systems.

Generalized synchronization, which extends the identical synchronization phenomenon and implies the presence of some functional relation between two coupled systems; if this relationship is the identity, we recover the identical synchronization.

Phase synchronization, which means driving of phases of chaotic oscillators, whereas their amplitudes remain uncorrelated. Lag synchronization, which appears as a coincidence of shifted-in-time states of two systems.

In this article, the identical synchronization is investigated in a complete network which means that each node connects to all other nodes of network [2], [3]. For example, Figure 1 shows the complete graphs from 2 to 10 nodes and complete graphs of 40 nodes. In this study, each node represents a neuron modeled by a system of ordinary differential equations on FHN type and each edge represents a synaptical connection modeled by a coupling function.

Definition 1. *Let* $S_i = (u_i, v_i),\ i = 1, 2, ..., n$ *and* $S = (S_1, S_2, ..., S_n)$ *be a network. We say that* S *synchronizes identically if* $\lim_{t\to+\infty}\left|u_j - u_i\right| = 0$ *and* $\lim_{t\to+\infty}\left|v_j - v_i\right| = 0,$ *for all* $i, j = 1, 2, ..., n.$

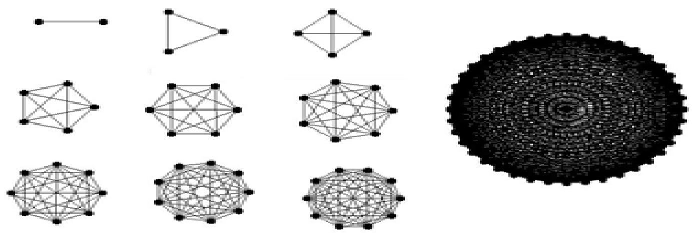


Figure1. *Complete graphs from 2 to 10 nodes and complete graphs of 40 nodes. In our study, each node represents a neuron modeled by a system of ordinary differential equations of FHN type and each edge represents a synaptical connection modeled by a coupling function.*

A system of n "neurons" (1) bi-directionally coupled by the electrical synapses, based on FHN, is given as follows:

$$\begin{cases} \varepsilon u_{it} = f(u_i) - v_i - g_n \sum\limits_{j=1, j\neq i}^{n} (u_i - u_j) \\ v_{it} = a u_i - b v_i + c \end{cases} \qquad i = 1, 2, ..., n, \tag{4}$$

where g_n is the coupling strength between u_i and u_j.

Theorem 1. Suppose that g_n $g_n > \frac{M}{n}$

where $M = \sup\limits_{u\in B, x\in\mathbb{R}} \sum\limits_{k=1}^{3} \frac{f^{(k)}(u)}{k!} x^{k-1}$, *B is a compact interval including u and* $f^{(k)}(u)$ *is the k*-th *derivative of f with respect to* u. *Then the network (4) synchronizes in the sense of Definition 1.*

Proof. Let $\Phi(t) = \frac{1}{2}\left[\sum\limits_{i=2}^{n}\left(a\varepsilon(u_i - u_1)^2 + (v_i - v_1)^2\right)\right].$

By deriving the function $\Phi(t)$, there is the following:

$$\frac{d\Phi(t)}{dt} = \sum_{i=2}^{n}\left[a\varepsilon(u_i - u_1)(u_{it} - u_{1t}) + (v_i - v_1)(v_{it} - v_{1t})\right]$$

$$= \sum_{i=2}^{n}\left[a(u_i - u_1)\left(f(u_i) - v_i - g_n \sum_{k=1,k\neq i}^{n}(u_i - u_k) - f(u_1) + v_1 + g_n \sum_{l=2}^{n}(u_1 - u_l)\right) + (v_i - v_1)\left(a(u_i - u_1) - b(v_i - v_1)\right)\right]$$

$$\le \sum_{i=2}^{n}\left[a(u_i - u_1)\left(f(u_i) - f(u_1) - ng_n(u_i - u_1)\right) - b(v_i - v_1)^2\right]$$

$$\le \sum_{i=2}^{n}\left[a(u_i - u_1)^2\left(f'(u_1) - ng_n + \sum_{k=2}^{3}\frac{f^{(k)}(u_1)}{k!}(u_i - u_1)^{k-1}\right) - b(v_i - v_1)^2\right]$$

$$\le \sum_{i=2}^{n}\left[a(u_i - u_1)^2 (M - ng_n) - b(v_i - v_1)^2\right].$$

If $g_n > \frac{M}{n}$, then $\frac{d\Phi(t)}{dt} \le -\beta\Phi(t) \Rightarrow \Phi(t) \le \Phi(0)e^{-\beta t}$,

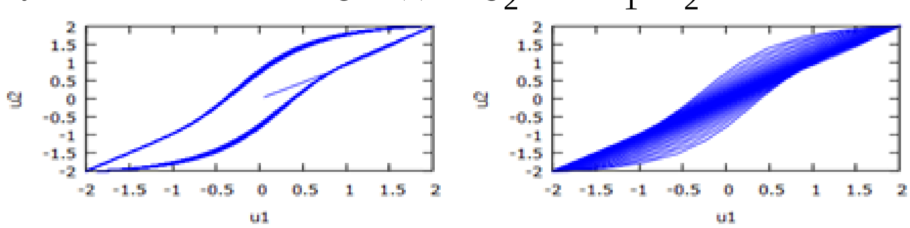
where $\beta = \min\left(2\frac{ng_n - M}{\varepsilon}, 2b\right)$. Thus, the synchronization occurs if the coupling strength verifies $g_n > \frac{M}{n}$. In the case where f is cubic, there is the following corollary.

Corollary 1. *Suppose that if is a cubic function,* $f(u) = m_3u^3 + m_2u^2 + m_1u + m_0$, *where* m_3, m_2, m_1, m_0 *are constants with* $m_3 < 0$ *and if* $g_n > \frac{1}{n}\left(m_1 - \frac{m_2^2}{3m_3}\right)$, *the network* $S = \left((u_1, v_1), (u_2, v_2), ..., (u_n, v_n)\right)$ *synchronizes in the sense of Definition 1.*

3. Numerical simulations

In the following part, the paper shows the numerical results obtained by integrating the system (4) where $n = 2, f(u) = -u^3 + 3u$, and with the following parameter values: $a = 1, b = 0.001, c = 0, \ \varepsilon = 0.1$. The integration of system was realized by using C++.

Figure 2 illustrates the phenomenon of synchronization. The simulations show that the system synchronizes from the value $g_2 = 1.4$. The figures (a), (b), (c), (d) represent the phase portraits (u_1, u_2) corresponding to the different values of coupling strength. It is easy to see that the synchronization occurs in figure (d) for $g_2 = 1.4$, $u_1 \approx u_2$.



(a) (b)

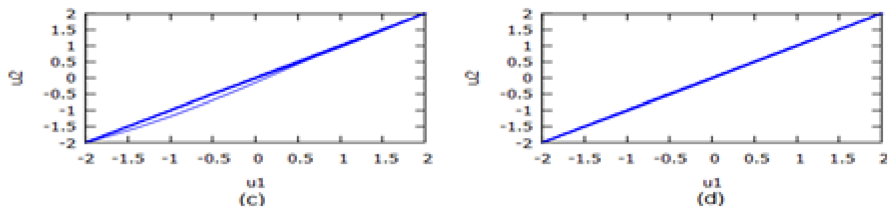


Figure 2. *Synchronization of a complete network of two linearly coupled "neurons" with* $f(u) = -u^3 + 3u,\ \ a = 1, b = 0.001, c = 0, \varepsilon = 0.1$. *The synchronization occurs for* $g_2 = 1.4$. *Before synchronization, for* $g_2 = 0.0001$, *the figure (a) represents the temporal dynamic of* u_2 *with respect to* u_1 *; the figure (b) represents the temporal dynamic of* u_2 *with respect to* u_1 *for* $g_2 = 0.01$*; the figure (c) represents the temporal dynamic of* u_2 *with respect to* u_1 *for* $g_2 = 0.5$. *For the value* $g_2 = 1.4$, *the synchronization of two "neurons" occurs:* $u_1 \approx u_2$

The following research focuses on the minimal values of coupling strength g_n to observe a phenomenon of synchronization between *n* subsystems modeling the function of neuron.

From the above result, in the case of two linearly coupled neurons, for the coupling strength which is bigger than or equal to $g_2 = 1.4$, these neurons have a synchronous behavior (figure 2). By doing similarly for the complete networks of n linearly identical coupled neurons, the values of coupling strength are reported in table 1.

Table 1. *The minimal coupling strength necessary to observe a phenomenon of synchronization of n linearly coupled neurons*

n		2	3	4	5	6	7	8	9	10
g_n		1.4	0.933	0.7	0.56	0.467	0.4	0.35	0.311	0.28
n	11	12	13	14	15	16	17	18	19	20
g_n	0.255	0.233	0.215	0.2	0.187	0.175	0.165	0.156	0.147	0.14
n	21	22	23	24	25	26	27	28	29	30
g_n	0.133	0.127	0.122	0.117	0.112	0.108	0.104	0.1	0.097	0.093
n	31	32	33	34	35	36	37	38	39	40
g_n	0.09	0.088	0.085	0.082	0.08	0.079	0.076	0.074	0.072	0.07

Following these numerical experiments, it is easy to see that the coupling strength required to observe the synchronization of *n* neurons depends on the number of neurons.

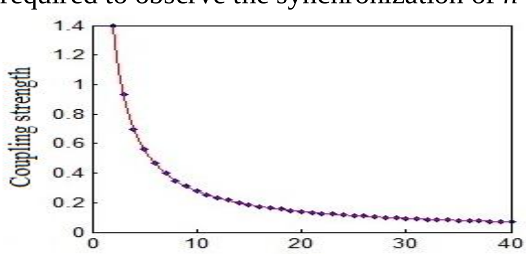


Figure 3. *The evolution of the coupling strength* g_n *for which the synchronization of n neurons takes place according to the number n linearly coupled neurons in complete network, and it follows the law* $g_n = \dfrac{2g_2}{n-1}$

Indeed, the points in figure 3 represent the coupling strength of synchronization according to the number of neurons in complete network, and the red curve represents the representative one: $g_n = \dfrac{2g_2}{n-1}$, where *n* is the number of neurons in network and g_2 is the coupling strength that permits to have the synchronization of two neurons coupled. Thus, the coupling strength necessary to obtain the synchronization of *n* neurons follows this law.

4. Conclusion

The paper shows a phenomenon of synchronization in complete network of *n* coupled systems of ordinary differential equations on Fitzhugh-Nagumo type. From Theorem 1, there is the result $g_n > \frac{M}{n}$ which shows that g_n becomes smaller when n takes the big values. Numerically, it is seen that the synchronization is stable when the coupling strength is exceeded to certain threshold and depends on the number of "neurons" in graphs. The bigger the number of "neurons" is, the easier we obtain the phenomenon of synchronization. Then, there is a compromise between the theoretical and numerical results. For future works, it is interesting to study about the identical synchronization in complete network coupled by chemical synapse.

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RAPID IN VITRO PROPAGATION OF NATIVE *DENDROBIUM CHRYSOTOXUM*

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Received: 23 April 2018/ Accepted: 11 June 2019/ Published: June 2019

Abstract: *The rapid micropropagation of Dendrobium chrysotoxum by applying in vitro tissue culture has the following results: The VW containing 20g/l sucrose and 8g/l agar was the most suitable medium for seed germination. The appropriate medium for rapid protocorm multiplication and shoot formation was the basic MS supplemented with 20g/l sucrose and 8g/l agar. The basic MS containing 20g/l sucrose, 8g/l agar, TDZ 4μM and NAA 1μM was the most optimum medium for shoot propagation from seedlings and gave the best shoot quality. NAA had little effect on generating roots of Dendrobium chrysotoxum in vitro.*

Keywords: *Dendrobium chrysotoxum, TDZ, NAA, protocorm, shoot.*

1. Introduction

The family Orchidaceae is one of the largest families in the world with more than 35,000 species scattering throughout the earth [1], in which *Dendrobiums* include about 1400 species [5]. *Dendrobium chrysotoxum*, known as the "Gold Orchid", is the popular and favorite species due to their large, beautiful yellow flowers with sweet smell. *Dendrobiums* are normally reproduced asesxually by forming offshoots at a very slow rate in natural conditions [9]. The sexual reproduction of these orchids is difficult because their seeds are minute and have no endosperm (without nutrients). Consequently, they need symbiotic fungi in order to germinate. Therefore, tissue culture is the only method that can be used to produce large numbers of seedlings at low cost to meet the market demand and minimize the exploitation of wild orchids. There are many published studies about propagation of *Dendrobiums in vitro* such as *D. candidum* [11], *D. fimbriatum* [10], *D. nobile* [8], *D. tosaense* [6] and *D. densiflorum* [7]. However *in vitro* propagation of *Dendrobium chrysotoxum* is rarely mentioned [11]. The present study was carried out to establish an efficient *in vitro* propagation protocol for native *Dendrobium chrysotoxum* collected in Thanh Hoa province, Vietnam.

2. Materials and methods

2.1. Plant materials

7- month unripe green capsules of *Dendrobium chrysotoxum* were collected in the nature in Thanh Hoa province, Vietnam and used as explants to initiate culture.

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The surface of capsules was sterilized by totally dipping in 96% Ethanol for 5 minutes, then passing over the alcohol light. Each capsule was split open longtitudinally by using sterile scalpel to scoop out numerous minute exalbuminous seeds and diluted in 120ml of sterile distilled water. Seed suspension was spread on the surface of different medium.

Figure 1. *Dendrobium chrysotoxum and 7- month unripe green capsules*

2.2. Research methods

The experiments were carried out in the tissue culture laboratory. Samples were incubated in a culture room at 25 ± 2^oC for 10 hour photoperiod provided by white fluorescent light of 2,300 lux intensity. The pH of medium was adjusted to 5.8 prior to sterilizing at 121^oC at 1atm for 20 minutes. The experiments were arranged according to the normal methods of tissue culture, following the Randomized Complete Block Design with 3 replicates/ treatment (10 Erlenmayer flasks/ replicate).

Seeds were sowed on three types of media: Vacin and Went 1949 (VW), KnudsonC 1965 (KC), Murashige-Skoog 1962 (MS) to determine the suitable medium for seed germination and protocorm generation (1^{st} Experiment). Good protocorms inducted from the 1^{st} Experiment (Ex) were transplanted into different media (MS, VW and KC) to study the effect of these media on the multiplying ability of protocorms (2^{nd} Experiment). Six- week good seedlings inducted from optimum medium of the 2^{nd} Ex were transplanted on the base medium (the optimum medium resulted from the 2^{nd} Ex) supplemented with TDZ *(Thidiazuron or 1-phenyl-3-(1,2,3 thiadiazol-5-yl) urea)* ranging from 0-6 μM and NAA ranged from 0-3 μM to evaluate the effects of TDZ and NAA on growth and development of *Dendrobium chrysotoxum*'s shoots and seedlings. Measures for monitoring and evaluating the results were carried out according to normal methods of tissue culture. Data obtained from the experiments was analysed by using Microsoft Excel and 5.0 IRRISTAT software.

3. Results and discussion

3.1. Effect of medium on seed germination, protocorm generation and seedling formation of Dendrobium chrysotoxum

Seeds germinated on all three types of media. When sowing seeds on VW, seeds germinated earliest (1.43 weeks) at 100% success rate and the highest percentage of protocorm and seedling emergence rate (100 % and 90% respectively). On MS medium, seeds germinated 4 days later than those on VW with low percentage of seedlings (only about 30% of 50% germinated seeds).

The results of this experiment were consistent with the statement of Kauth *et al* (2008) about the effect of ammonium salt on germination of orchid seeds [4]. Ammonium salt is essential for germination of orchid seeds. However ammonium salts inhibit germination of some orchid species seeds such as *Dactylorhiza incarnata*, *Vanda tricolor*. Our experiment showed that the germination initiation and germination percentage of the seeds decreased significantly with the increase of the ammonium concentrations (Ammonium concentration on VW, MS and KC is 7.57 mM, 10.31 mM and 13.82 mM respectively).

Although 30% of the orchid seeds germinated into the dark green color sample on KC medium, the sample was not able to form protocorms and was completely dead after 10 weeks of culture (Fig 2). In conclusion, the suitable medium for seed germination of *Dendrobium chrysotoxum* is VW medium.

Table 1. *Effect of medium on seed germination, protocorm generation and seedling formation*

Trea-ment	Media	Initiation of germi-nation (Weeks)	Seed germi-nation percen-tage (%)	Develop-ment of protocorm (Weeks)	Protocorm percentage (%)	Initiation of the 1st two leaves (Weeks)	Seedling percen-tage (%)
1	MS	2,0	30,25 (+)	7,14	50,0 (++)	14,20	30,50 (++)
2	VW	1,43	100 (+++)	6,0	100 (+++)	12,13	90,25 (+++)
3	KC	3,14	30,22 (+++)	_	0	_	

Note: "_" not initiation; "+" Bad quality, light green; "++" Medium quality, green; "+++" Good quality, dark green.

Figure 2. *Effect of media on seed germination, protocorm generation and seedling formation (from the left to the right MS, KC và VW)*

3.2. Effect of culture medium on protocorm multiplication and shoot initiation

The 8-week protocorms (from sowing) obtained on VW from the 1st Ex were transferred to different media including MS, VW and KC to determine the effect of medium on protocorm multiplication. The results obtained after 4 weeks are presented in Table 2.

Table 2. *Effect of media on protocorm multiplication and shoot initiation 4 weeks after the first transplantation*

Treatment	Medium	Percentage of samples formed protocorm (%)	Protocorm multiplication	Shoot percentage (%)
1	MS	100	+++ Dark green	97,13
2	VW	100	++ Light green	50,35
3	KC	100	+ Light green	0

Note: "+" Slow multiplication; "++" Medium multiplication; "+++"Fast multiplication

The transplanted protocorms to three media were capable of producing new protocorms. On MS, protocorms had the highest multiplication and shoot emergence with 97.13%. Protocorms and shoots on MS were very good and had a dark green color. On VW, the percentage of shoot emergence was only 50.35% with the light green color. The protocorms initially created new protocorms with very slow rates and stopped completely after two weeks of culturing on KC. These protocorms were not able to form shoot buds (Fig 3).

The results showed that protocorms and shoots occurred respectively on all media. Shoot buds only initiated as the formation of new protocorms has slowed or stopped. VW was the optimal medium for seed germination but was not the suitable medium for protocorm propagation. MS was the suitable medium for rapid protocorm multiplication and budding of *Dendrobium chrysotoxum*. KC medium was not recommended for this orchid species.

Figure 3. *Effect of culture medium on protocorm multiplication and shoot initiation after 4 weeks of the first transplantation (from the left to the right MS, KC and VW)*

3.3. Effect of TDZ and NAA on the shoot initiation and the development of seedlings

Many authors concluded that the *in vitro* differentiation of plant organs is the interaction of auxin and cytokinin group. High incidence of auxin/cytokinin will stimulate rooting. On the other hand, low incidence will promote the differentiation of shoots [3]. The results of the study on the effect of TDZ and NAA on shoot initiation and development of seedlings are presented in Table 3.

Table 3. *Effect of TDZ and NAA on shoot initiation and the development of seedlings after 4-week culturing of seedlings taken from the 2nd transplantation (*)*

Treatment	Concentration (µM)		Number of leaves/ explant	Number of roots/ explant	Shoot initiation percentage (%)	Coefficient of shoot multiplication (times)
	TDZ	NAA				
CT1(Control)	0	0	2,17 **a**	2,25 **a**	0,0	1,00 **a**
CT2	2	0	5,00 **b**	0,67 **b**	80,3	2,10 **b**
CT3	4	0	5,14 **b**	0,24 **c**	88,25	2,29 **c**
CT4	6	0	4,15 **c**	0,00 **d**	75,52	1,93 **d**
CT5	0	1	4,08 **c**	0,83 **e**	17,34	1,58 **e**
CT6	2	1	5,13 **b**	0,73 **be**	83,02	2,17 **bc**
CT7	4	1	5,92 **d**	0,00 **d**	100	2,42 **c**
CT8	6	1	4,46 **e**	0,00 **d**	83,23	2,0 **bd**
CT9	0	3	3,72 **f**	1,11 **f**	33,06	1,5 **e**
CT10	2	3	4,54 **e**	0,89 **e**	86,25	2,11 **b**
CT11	4	3	5,54 **g**	0,79 **be**	88,12	2,29 **c**
CT12	6	3	4,75 **e**	0,65 **b**	79,41	2,1 **b**
LSD $_{0,05}$			0,224	0,157		0,146

Note: Means within collumn followed by different letters are significantly different (p= 0.05) based on LSD0,05. (*) 1st transplantation was carried out after 8 weeks since sowing. 2nd transplantation was carried out after 6 weeks since the 1st transplantation.

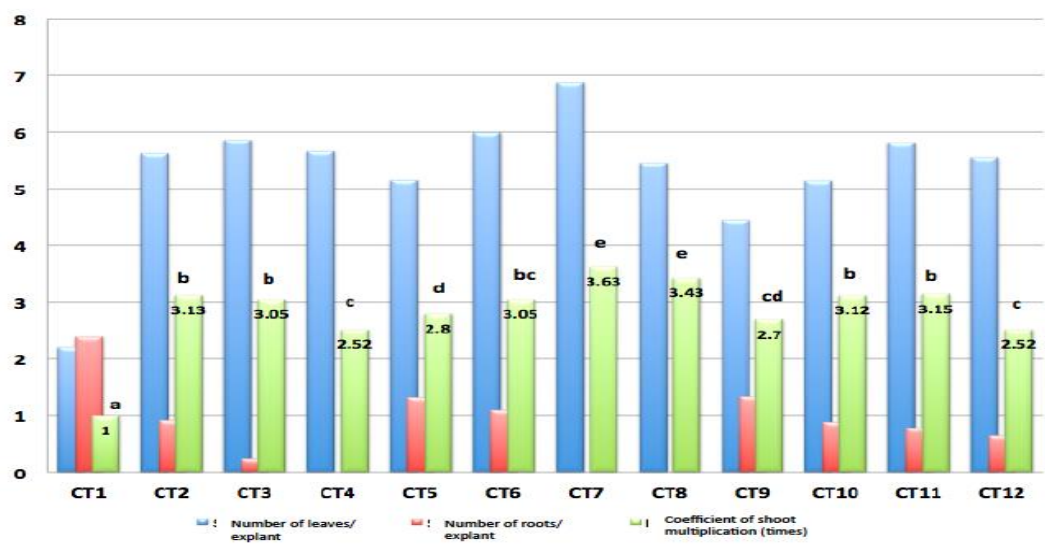

Figure 4. *Effect of TDZ and NAA on shoot initiation and the development of seedlings after 8- week culturing of seedlings taken from the 2nd transplantation*

3.3.1. Effect of TDZ and NAA on shoot initiation

Shoot multiplication coefficient of *Dendrobium chrysotoxum* after 4-week culturing was the highest at TDZ 4µM in combination with 0 or 1 or 3µM NAA. After 8 weeks of culturing, shoot multiplication was the highest at TDZ 4µM + NAA 1µM and TDZ 6µM + NAA 1µM. Thus on the TDZ 4µM + NAA 1µM medium, seedlings had stabler and better shoot multiplication than those on the other media. In the control treatment, all seedlings were not able to produce buds and had morphological variations that were significantly different from those of the other treatments. After 3 weeks of culturing, seedlings on this medium had pseudobulbs. This was the reason that these plants were less likely to produce buds than plants in other treatments (Fig.5).

3.3.2. Effect of TDZ and NAA on rooting

D. chrysotoxum plants rooted best on the medium without growth regulator, followed by those on NAA 3µM supplemented medium (Fig.5). Plants in other media had lower rooting ability. Especially high shoot multiplication plants had no roots (Fig.5).

According to Gantait *et al* (2009) [2], different auxins have different effects on root formation of orchids. This effect is probably due to the affinity of auxin receptors when involved in root formation. For *D. chrysotoxum*, Gantait claimed that IBA has much greater positive effect on root formation than other auxins including NAA. This conclusion was also drawn from the experiments of Sreckumar *et al* (2000) about *Hermidesmus indicus* [13]. Our results were consistent with previous research that NAA had little effect on the rooting ability of *D. chrysotoxum*. Therefore, the most suitable medium for rooting was the one without TDZ as well as NAA.

3.3.3. Effect of TDZ and NAA on leaf initiation

In treatments with TDZ 4µM + NAA 1µM as well as TDZ 4µM + NAA 3µM, the plants had the highest number of leaves compared to those in other treatments. This can be easily explained because these plants had the highest shoot multiplication coefficient which resulted in stronger leaf initiation (Fig 5). Plants in Control had the least number of leaves (2.17 leaves /tree in average) (Fig.5).

Figure 5. *4- week seedlings from seedlings obtained from 2nd transplantation on medium with TDZ 4µM + NAA 1µM; Control; and TDZ 0.0µM + NAA 3.0µM (from the left to the right)*

4. Conclusion

The VW contained 20g/l sucrose and 8g/l agar was the most suitable medium for seed germination *in vitro*.

The appropriate medium for rapid protocorm multiplication and shoot formation was the basic MS supplemented with 20g/l sucrose and 8g/l agar.

The basic MS contained 20g/l sucrose, 8g/l agar, TDZ 4µM and NAA 1µM was the most optimum medium for shoot propagation from seedlings and gave the best shoot quality.

NAA had little effect on generating roots of *Dendrobium chrysotoxum* in *in vitro* condition.

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LAND POTENTIAL PRODUCTIVITY ASSESSMENT FOR ANNUAL AGRICULTURAL CROPS DEVELOPMENT IN QUANG XUONG DISTRICT BY APPLYING MULTI-CRITERIA EVALUATION AND GIS

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Received: 8 June 2018/ Accepted: 11 June 2019/ Published: June 2019

Abstract: *The assessment of land potential productivity for agricultural production and land suitability for selected annual crops was based on FAO guidelines for land evaluation which were adopted and slightly modified for compatibility with Vietnamese conditions. A combination of three main factors consisting of ten variables of all related data were stored, analyzed, mapped and presented in ArcGIS software. Weighted Linear Combination Method developed by Hopkins and GIS techniques were used to analyze and determine the land potential for agricultural use in the study area. The results show that 5.26%, 83.10%, 10.06%, and 1.57% of the investigated areas were assessed as high potential, moderate potential, low potential and very low potential, respectively for growing crops. The findings from this study were also useful to support the land users and land managers to exploit agricultural land effectively.*

Keywords: *Land suitability, GIS, Land potential.*

1. Introduction

In the process of land evaluation, a scientifically standardized technique is used to estimate the characteristics of land resources for certain uses and its results can be used as a guidance for land users and planners to find out a better use [10]. As a land is a limited resource, reliable and accurate land evaluation is indispensable for assisting decision-makers and land users to use the scarce land resources efficiently and develop models to predict the land suitability for different types of agriculture [6].

In terms of physical potential, land suitability evaluation tries to find the best places of the land that suit a given range of utilization types, which may be included agricultural uses or nature preservation alternatives as well. The procedure hereby in use is based on the crop requirements for growth and the environmental conditions [1], [11]. Evaluation of land suitability is analyzing the criteria from different land resources and socio-economic conditions [8].

GIS and MCE (Multi-Criteria Evaluation) are capable of assisting land users in making a right decision in that GIS effectively controls assessment factors and MCE synthesizes them into a suitability index [7]. The combination between MCE and GIS techniques is both traditional and modern approaches to analyzing land evaluation, primarily aiming at evaluating factors and recommending feasible decisions [9].

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Agriculture is one of the most important economic sectors in Quang Xuong District. It occupied more than 50% of the entire area. However, the cultivated lands have been decreasing over years because of the population growth and the demand for expansion of build-up areas and rural infrastructure development. Thus, appropriate land use planning will be the best way to increase agricultural yield as well as protect cultivated land. Potential productivity assessment is a prerequisite step of evaluating whether or not a specific land is suitable for development of sustainable agriculture.

This study presents the results of spatial analysis to specify potential areas for agricultural production with GIS techniques based on the FAO guideline [2], [3], [4] for land evaluation.

2. Study area

The topography of Quang Xuong district is relatively flat, and runs from the north to the south. The climate condition of Quang Xuong is affected by the tropical and temperate zone. It is hot and humid weather by influence of the south-westernly dry wind in the summer; dry and with little rain, occasional appearance of frost in the winter. The average temperature per day is about 23^0C. The annual average precipitation ranges from 1600mm to 2000mm and is irregularly distributed. The humidity is rather high, the average account is over 80% in most of the months and is rarely under 60%.

According to soil classification methods of FAO-UNESCO, the study area has 6 major soil groups, with 12 soil units and 18 sub-units. This is a principle for expecting feasible results of various agricultural crops.The average characteristics of the major soil groups is presented in Table 1 and the soil texture of sub-units of soil are presented in Figure 1.

Table 1. *Fertility of soil group in Quang Xuong District*

Soil group	pH_{KCL}	Average OM (%)	Total concentration (meq/100g)		Available concentration (meq/100g)		Exchange cation (meq/100g)	CEC (meq/100g)	BS (%)
			P_2O_5	K_2O	P_2O_5	K_2O			
Arenosols	4.90	0.35	0.035	0.50	16.25	5.40	4.40	6.20	38.0 – 83.0
Salic Fluvisols	5.45	2.70	0.09	1.43	18.50	9.50	10.50	1.50	78.0 – 87.0
Fluvisols	4.90	2.50	0.08	1.05	14.50	7.25	8.40	15.50	26.0 – 86.0
Gleysols	5.05	1.85	0.054	1.72	11.75	4.75	8.50	15.00	45.0 – 84.0
Acrisols	4.30	0.60	0.035	0.51	3.25	6.20	4.00	8.50	35.0 – 57.0
Leptosols	4.56	2.14	0.12	2.30	7.0	21.02	8.60	16.88	28.0 – 50.0

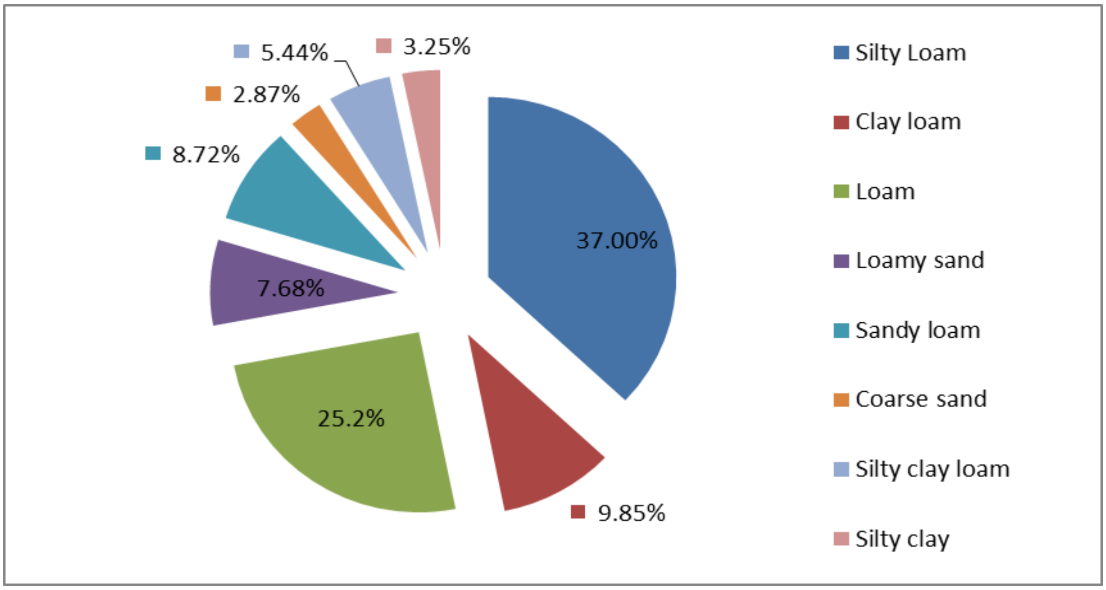

Figure 1. *The soil texture for sub-units of soil in the Quang Xuong District*

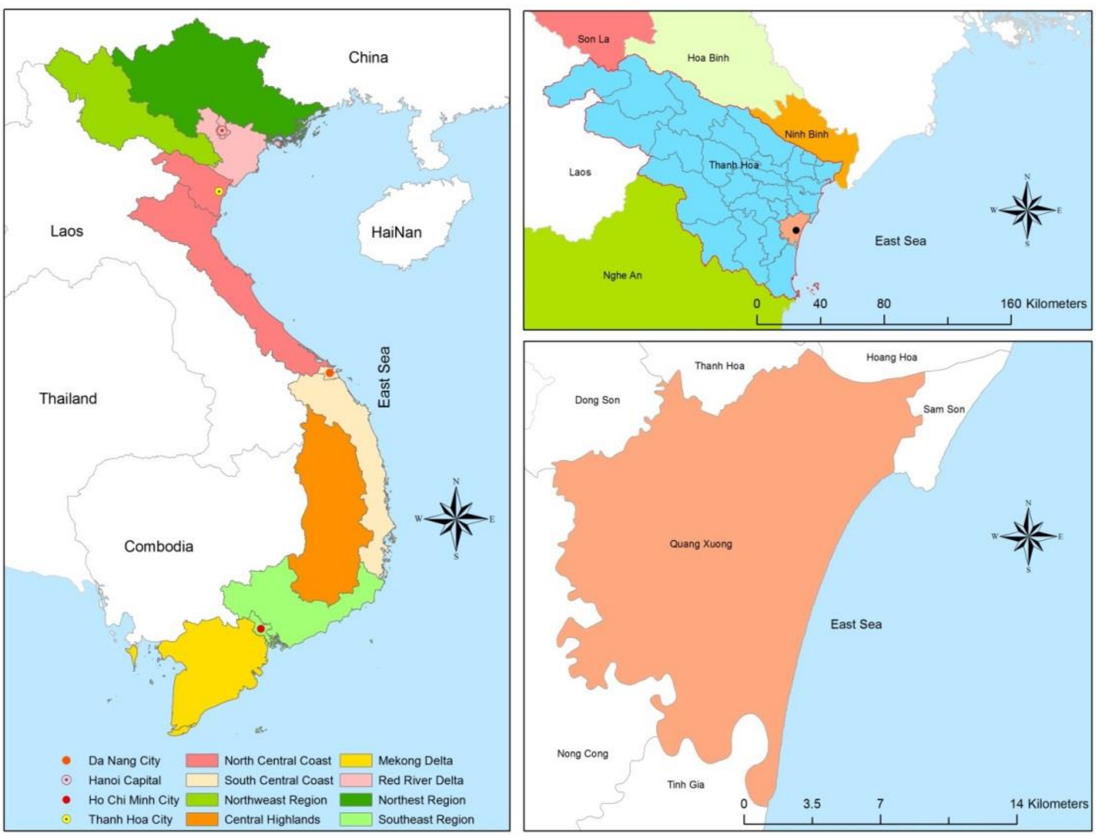

Figure 2. *Location and boundary of the study area*

Source: Department of Natural Resources and Environment Management of Thanh Hoa Province

3. Methodology

At present, the study area does not have a standard model of land capability evaluation for agricultural use. Thus, we examine and generate a potential productivity map for agricultural production in the district based on the available data and the requirement for farming production. The Linear Combination Method developed by Hopkins [5] with the help of GIS was used to express land potential for agricultural production.

3.1. Selection of factors, variables and database development

Three main components and ten variables have been chosen for assessment of land capability (Table 2), namely: (1) chemical property factor including organic matter (OM); cation exchange capacity (CEC); pH; sum of exchangeable basic cation (EC); and base saturation (BS), (2) physical property involving soil texture; irrigated condition; soil depth; and drainage capacity, (3) relative topography including depressed; low flat; flat; upper flat; and high topography. These factors and variables are differently dependent on land capability productivity. The database was developed by using ArcGIS software.

Table 2. *Main factors and their variables for land potential assessment*

Main factor	Variables	Units
Chemical property	OM	-
	CEC	meq/100g soil
	Soil pH	-
	Exchangeable cation	meq/100g soil
	Base saturation	%
Physical property	Soil texture	-
	Irrigated condition	-
	Soil depth	Cm
	Drainage capacity	-
Relative topography	Relative Topography	-

3.2. Land potential productivity model

The potential levels for agricultural production were dependent on the score distribution of each site. The final score of the land capability was calculated by the formula (1) and converted to a level of capability as described in table 3.

$$Score = \frac{\sum_{i}^{n} \mathrm{W}iXxyi}{\sum_{i}^{n} \mathrm{W}i} \qquad (1)$$

Where: n is the number of factors, W_i is the weight of factor i, X_{xyi} is the score of category for each variable of each factor i.

Table 3. *The level of land potential productivity for annual agricultural crops development*

Score	Potential capacity	Description
≥ 3.5	Highly potential productivity	The land has few limitations for agricultural production, its potential productivity is high.
2.5 - 3.5	Moderately potential productivity	The land has some limitations for agricultural production, its potential productivity is medium.
1.5 - 2.5	Lowly potential productivity	The land has a number of serious limitations for agricultural production, its potential productivity is low.
≤ 1.5	Very low potential productivity	The land has a large of serious limitations for agricultural production, its potential productivity is very low.

4. Results and discussion

4.1. Determination of weights of main factors and variables

In this research, the weights were determined from an average value based on the result of interviewing people who have experience and knowledge in the agricultural field (Table 4). The score of each variable category associated with requirements of potential productivity levels was defined by discussing with local officers. Based on the result of the discussion, the ranking of each variable was clarified from 1 to 4, with 1 being the worst for agricultural use and 4 the best. These are very low potential, low potential, medium potential and high potential corresponding to Arabic numerals of 1, 2, 3 and 4 (Table 5).

Table 4. *Weight of each factor and variable*

Main factor	Weight 1 (%)	Variables	Unit	Weight 2 (%)	Overall Weight (%)
Chemical properties	40	OM	-	30	12
		CEC	meq/100g soil	25	10
		Soil pH	-	25	10
		EC	meq/100g soil	10	4
		BS	%	10	4
Physical properties	35	Soil texture	-	30	10.5
		Irrigated condition	-	30	10.5
		Soil depth	cm	25	8.8
		Drainage capacity	-	15	5.2
Topography	25	Relative Topography	-	-	25

Table 5. *Score of each variable category for land potential productivity assessment*

Variable	Category	Score	Variable	Category	Score
Base saturation	> 50%	4	Soil texture	Silty loam, Loam, Silty clay loam	4
	35% - 50 %	3		Silty clay, Clay loam	3
	< 35 %	1		Loamy sand, Sandy loam	2
				Coarse sand	1
OM (%)	> 2	4	Irrigation	Actively irrigated	4
	2 - 1.5	3		Somewhat irrigated	3
	1.5 - 0.8	2		Poorly irrigated	2
	< 0.8	1		None irrigated	1
CEC (meq/100g soil)	> 15	4	Soil depth	> 70cm	4
	15 – 10	3			
	< 10	1		50cm – 70cm	3
pH	> 6.5 - < 7.0	4		30cm – 50cm	2
	6.5 - 6.0	3		< 30cm	1
	6.0 - 5.5	2	Drainage	Good	4
	< 5.5	1		Moderate	2
			Relative topography	Flat	4
EC (meq/100g soil)	> 8.0	4		Low flat	3
	8.0 - 4.0	3		Upper flat	3
	< 4.0	1		High	2
				Depression	2

4.2. Assessment of chemical factors

Chemical properties of soil include five variables (OM, CEC, pH, EC, BS) as determinants of agricultural land quality such as agricultural productivity. It is commonly regarded as an important predictor of potential productivity of farmlands. The results of chemical factor examination for agricultural potential use are presented in Table 6. Based on the table, 1464.57ha of cultivated land is classified under high potential level, accounting for 10.50% of the research area and only located on the Fluvisols group. 5797.51ha is assessed as moderate potential category, accounting for 42.89%, prevailing in Fluvisols, Gleysol, and Arenosols groups. The low potential level is about 5293.80ha, occupying for 37.97% and is distributed in the Fluvisols, Gleysols, salicFluvisols and Arenosols. The area for very low potential of agricultural use is about 1203.93ha or 8.64% of the entire investigated area and mainly distributed in the Acrisols, Arenosols, and Leptosols groups.

Table 6. *Potential productivity level of chemical factor for annual cultivation*

Land use purpose	Potential level	Area (ha)	Percent (%)
Annual cultivation	High potential	1464.57	10.50
	Moderate potential	5979.51	42.89
	Low potential	5293.80	37.97
	Very low potential	1203.93	8.64
Sum		**13,941.81**	**100**

4.3. Assessment of physical factor

The potential productivity was the result of overlaying thematic maps of soil texture, soil depth, irrigation condition, and drainage capacity. The details of physical factor assessment for cultivation use are described in Table 7. Based on the results examination, there is no agricultural area under the very low potential situation in the study area, and most of the investigated land is of moderate capability for agricultural development with 11,152.26ha, covering 79.99%. The areas with low potential productivity amounted to a small proportion compared with entire area for agricultural use. They cover about 619.74ha, equivalent to 4.45%. The results also demonstrate that the land with high potential productivity for crops growth is about 2169.81ha, occupying 15.56% of the total examining area. In general, the research area has a good condition of physical properties for agricultural development.

Table 7. *Potential productivity level of physical factor for annual cultivation*

Land use purpose	Potential level	Area (ha)	Percent (%)
Annual cultivation	High potential	2169.81	15.56
	Moderate potential	11,152.26	79.99
	Low potential	619.74	4.45
Sum		**13,941.81**	**100**

4.4. Assessment of relative topographic factor

The study area is a coastal sandy land, so most of the areas are plain except for an area of 219.33ha of the Leptosols group whose slope is more than 25^0 and considered as unsuitable for annual agricultural crops. In Vietnam, the term of relative topography is usually used in land evaluation projects for the plains at districts, communes, villages or small areas. Based on the natural condition of the area, the observation, experts' opinions, and discussion with local farmers, the topography was classified into five classes as flat, low flat, upper flat, depression and high. The potential levels of classification are shown in Table 8. This factor of the enquired area is assessed as follows: high potential for cultivated activities is 1785.06ha or 12.80%; moderate potential is 8699.13ha, covering of 62.40%, 2132.10ha of which belongs to low flat, and 6567.03ha is topography of upper flat. The total assessed area of low potential is 3328.29ha or 23.23%, 2837.88ha of which is under the relative topography of depression, and 400.41ha is under the highly topographic condition.

Table 8. *Potential level of relative topographic factor for annual cultivation*

Land use purpose	**Potential level**	**Relative topography**	**Area (ha)**	**Percent (%)**
Annual cultivation	High potential	Flat	1785.06	12.80
	Moderate potential	Low flat, upper flat	8699.13	62.40
	Low potential	Depression, high	3238.29	23.23
	Very low potential	Slope > 25^0	219.33	1.57
Sum			**13941.81**	**100**

4.5. Final land potential productivity assessment

The result of the multiplication of all the score points out which site is better for agricultural use based on the scores and weights expressed in the model. The weights for the other factors and the scores for its variables were calculated based on agricultural experts' opinions as well as local conditions.

Three main factors of current environmental conditions in the study area, including chemical soil property, physical soil property, and relative topography were overlaid together in one layer. The information about multiple overlays was input into GIS to find out land potential map for cultivation. Its results were mainly classified as high, moderate, low, and very low potential suitability level for agricultural use. The consequences of the classification indicate that 11585.97ha or 83.10% of the total investigated area is under low to medium potential for agricultural activities, while the smallest area with only 219.33ha, making up 1.57% was determined as very low potential productivity category and only concentrated on the Leptosols group with the soil depth of less than 30cm. The classification of land potential assessment is showed in Table 9 and Figure 3.

The results of land potential productivity assessment indicate that the highly potential level for agricultural production is only located in the Fluvisols group and with soil depth of more than 70cm and soil texture of silty clay loam. It was 733.77ha, covering 5.26% of the evaluated area. The results also show that the moderate potential level prevails in different types of soil groups such as the Fluvisols, salic Fluvisols, Acrisols, Arenosols, and Gleysols with different types of soil texture and the soil depths fluctuated from 50cm to 70cm. The low potential level for growing crops is mainly located in the Arenosols, Acrisols and a part of the Fluvisol groups with 1402.74ha, equivalent to 10.06%. It has different soil textures such as loam, clay loam and coarse sand with soil depth levels from 30cm to 50cm.

Table 9. *Land potential productivity assessment for annual agricultural crops*

Land use purpose	**Potential level**	**Area (ha)**	**Percent (%)**
Annual cultivation	High potential	733.77	5.26
	Moderate potential	11585.97	83.10
	Low potential	1402.74	10.06
	Very low potential	219.33	1.57
Sum		**13,941.81**	**100**

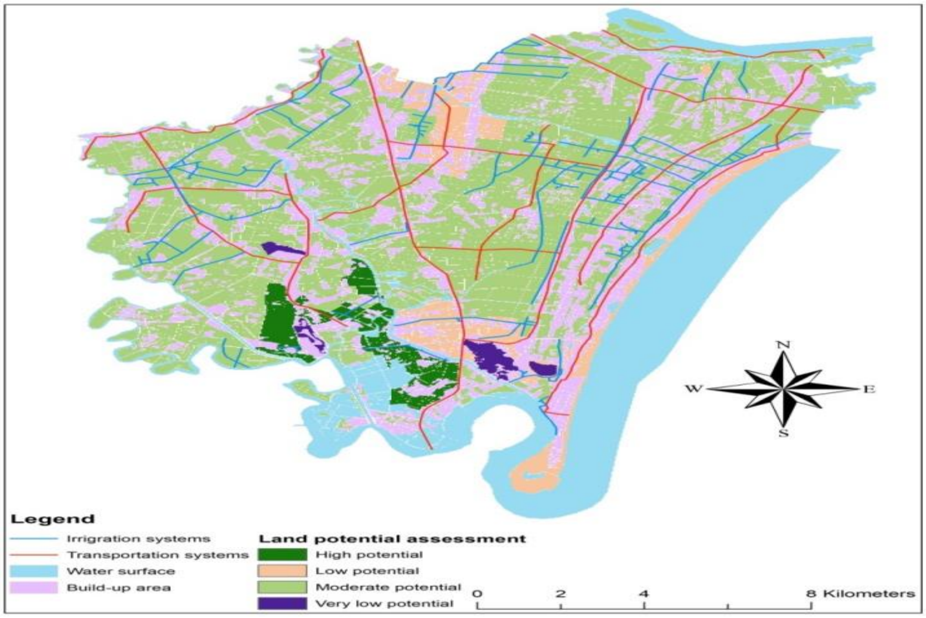

Figure 3. *Assessment of Land potential productivity for annual agricultural crops*

5. Conclusion

The results of defining potential areas for agricultural production using GIS techniques in combining with Weight Linear Combination Method [5] in this study are divided into four levels as highly potential, moderately potential, low potential, and very low potential productivity, respectively, in which, highly potential, moderately potential and low potential are considered as suitable for agricultural production, while very low potential is not suitable for croplands because of extremely severe limitations or hazards, but it can be used for permanent vegetation like forest or natural plant covering.

The findings from this study demonstrate that the process of determining the potential productivity is significantly useful to support the land users and land managers finding out the problems in a certain use of agriculture land and provided more information for appropriate investment for cultivated production. Furthermore, the results of land potential productivity assessment are also significant for local land users to make possible strategies for development of agricultural land in short term as well as in a long term of use. Finally, it is helpful for setting up a land information system for sustainable use of land resources and land management not only in the case study.

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USING FLIPPED LEARNING MODEL IN READING CLASSES TO DEVELOP ENGLISH MAJORS' READING COMPREHENSION

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Received: 3 July 2018/ Accepted: 11 June 2019/ Published: June 2019

Abstract: *This study focuses on examining the efficacy of flipped reading class model in developing English majors' reading comprehension. An experimental research was carried out in which the control group was taught with the traditional steps of a reading lesson while the experimental group was taught using flipped learning approach. A pretest and a posttest were administered to both groups before and after the experimental program to measure the effectiveness of flipped reading classes. A questionnaire, informal interviews and class observation were conducted to the experimental group to investigate into students' reaction to flipped reading lessons. The obtained results showed that flipped learning did help English majors develop their reading comprehension.*

Keywords: *Flipped learning, reading comprehension.*

1. Introduction

It was generally agreed that reading comprehension is one of the most crucial skills for foreign language learners in general and English language learners in particular to master. Reading in English has become a critical skill in terms of academic and career success [8]. With strengthened reading skills, English as a second/foreign language (ESL/EFL) readers will make greater progress and attain greater development in all academic areas [1]. Due to the great importance of reading comprehension in ESL/EFL learning, numerous studies have been conducted to facilitate effective reading lesson delivery.

However, our initial investigation into the current situation of teaching and learning foreign language reading skills at different universities revealed that reading classes seem rather boring with tedious activities in the course books. Moreover, it takes too much time for students, especially those who are learning advanced English reading courses, to read lengthened and complicated texts in class before they can do the required tasks.

This study is aimed at using flipped learning model in reading classes with the hope that English-majored students can develop their reading comprehension in a more active and interesting way. With flipped learning, tasks in the course books are done individually by students before class, and the valuable class time is reserved for active interaction between the teacher and students as well as between students and students themselves.

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2. Theoretical overview

2.1. Reading comprehension

According to John Kruidenier (2002) comprehension is an active process and the reader must interact and be engaged with the text for it to work well. As comprehension takes place, words are decoded and associated with their meaning in the reader's memory and phrases and sentences are processed rapidly or fluently enough so that the meanings derived from one word, phrase, or sentence are not lost before the text is processed. Snow (2002) defined reading comprehension "as the process of simultaneously extracting and constructing meaning through interaction and involvement with written language". He stated that comprehension entails three elements: the 'reader' who is doing the 'comprehending', the 'text' that is to be comprehended, and the 'activity' in which comprehension is a part.

Peter Westwood (2008) claimed that reading comprehension is often conceptualized as functioning at different levels of sophistication and referred to as literal, inferential and critical. The most basic level (literal) is where the reader is able to understand the factual information presented in a passage of text. The next level is referred to as the inferential level. At this level the reader is able to go beyond the words on the page and infer other details. Being able to operate at the inferential level means that the reader is using information effectively to deduce cause and effect, and to anticipate what may come next. At a more demanding level (critical reading), the reader is able to appraise what he or she is reading, detecting good writing style from the author, recognizing when some statements in the text are biased or incorrect, appreciating the writer's viewpoint, comparing and contrasting information with other facts they have read elsewhere, and reflecting upon the importance or otherwise of the opinions presented.

2.2. Flipped learning

The governing board and key leaders of the Flipped Learning Network (2014) have defined flipped learning as "a pedagogical approach in which direct instruction moves from the group learning space to the individual learning space, and the resulting group space is transformed into a dynamic, interactive learning environment where the educator guides students as they apply concepts and engage creatively in the subject matter".

In flipped classes, lessons/lectures are not taught during class time. They are delivered to students before class as input materials in the forms of screen casts, podcasts and videos. Students study by themselves at their own pace before class. Classroom time is for the students to apply knowledge by solving problems, doing practical work and getting necessary support. With the flipped classroom, students have direct access to the knowledge and the teacher serves as a coach and mentor. Students have to be well-prepared for their contact moments with their teacher and peers. During the class time teachers are to focus on knowledge application and deeper processing of the learning material.

However, flipped materials do not always have to be tied to technology [2]. Students can study various types of materials (e.g., readings from a textbook and worksheets developed by their teacher) on their own outside of class time and grasp the meaning of the content. Based on their understanding, they consolidate their content knowledge by raising questions and engaging in class activities through group work facilitated by their instructor [6].

3. The study

3.1. Subjects

Participants in the study consisted of two groups (27 students/1 group) of third-year English majors at Hong Duc University. One group served as the control group, and the other as the experimental group. These students were taking the final reading course in the curriculum. After finishing this course, students are expected to achieve English reading competency level equivalent to CAE.

3.2. Instruments

In order to find out whether this model worked well in reading classes and how the students reacted to the experimental program, different instruments were utilized in the study.

Pretest and posttest: A pretest was administered at the beginning of the course before the experimental program to make sure that two groups had the same level of reading comprehension. At the end of the course, a posttest was conducted to find out whether flipped learning made any difference to the reading comprehension of the experimental group in comparison with the control group. Both tests included questions to check students' reading comprehension at three different levels referred to as literal, inferential and critical.

Survey questionnaire: A small-scaled survey questionnaire was administered after the experimental program to investigate into students' reaction to the flipped classes.

Class observation: During the reading lessons, class observation was done by the researchers to see how students worked in the flipped classes.

Interviews: Interviews with the students in the experimental group were carried out to investigate into students' attitude to the experimental program.

3.3. Research methods

In order to measure the efficacy of the experimental program, both quantitative and qualitative methods were utilized.

Quantitative method: Results of the pretest and posttest as well as the survey questionnaire were synthesized and compared between the control and experimental groups to find out whether flipped learning could improve English majors' reading comprehension.

Qualitative method: Class observation and interviews with the students provided the researchers with raw data for analyzing the students' reaction and attitude to the experimental program.

3.4. Procedures

After the pre-test in the first week of the semester, a group was randomly chosen as the experimental group and the other as the control group. For the control group, three traditional steps of a reading lesson, namely pre-reading, while-reading and post-reading were carried out throughout the reading course.

For experimental group, the course begins with an introduction about flipped learning in which students read the reading materials and do all the reading comprehension tasks as required in the course book by themselves before going to class. Moreover, students had to read two more pieces of reading materials related to the topic of the lesson on the Reading

Group set up on the Messenger by the teacher at the beginning of the course. Then, they made comparison and found the connections among the three reading passages. Each student was required to make two questions about things that they wanted to know more about the topic of the lesson. Class time is for students to focus on deepening understanding, discussing issues related to the topics, seeking answers to their questions and applying knowledge in real life. All these were done with the help of in-class activities prepared by the teacher.

In a 50-minute flipped classroom, the teacher used the first 5 minutes to warm up the students as in traditional classes. The next 10 minutes is reserved for lexical comprehension: Understanding key vocabulary words in a text. Students are exposed to the important and new words in the text they have read before class, and attempt to use these words in meaningful sentences. Another 10 minutes is for literal comprehension: Answers the questions who, what, when, where, why, and how. The next 20 minutes is for inferential ability: Answers questions that have the reader relate the new information to background knowledge, deduce cause and effect, predict future events etc. The remaining 20 minutes is for critical reading: Comment on the writing styles, writers' bias, viewpoints, comparing and contrasting information in the three reading passages they have read before class etc. The experimental program took place in ten weeks from week 2 to week 11 of the semester. Class observation was conducted throughout the ten experimental weeks. The final week is reserved for the posttest. A survey questionnaire and informal interviews were carried out in the final week after the posttest.

3.5. Results and discussions

The results of the pretest and posttest administered at the beginning and at the end of the reading course to find out to what extent flipped learning model can help students improve their reading comprehension are presented in the following table.

Table 1. *Results of the pretest and posttest*

Points (/10)	Control group		Experimental group	
	Pretest (%)	Posttest (%)	Pretest (%)	Posttest (%)
8.0-10	7.41	7.41	11.11	14.81
6.5-7.5	25.93	33.33	22.22	59.26
5.0-6.0	62.96	55.56	59.26	25.93
0-4.5	3.70	3.70	7.41	0

As can be seen from the table, both the control and experimental groups have more or less the same level of reading comprehension in the pretest. However, after the experimental program, the experimental group witnessed better results with more students achieving 8-10 points. The number of the students who received 6.5-7.5 points also increased from 22.22% to 59.26%. Fewer students got mark 5-6 (with the number deceasing from 59.26% in the pretest to 25.93% in the posttest). No student got mark 0-4.5. In the meantime, the results of the control group experienced less change with the same number of students achieving excellent and weak marks in the pretest as in the post test. The number of the students who got mark 6.5-7.5 slightly increased from 25.93% to 33.33%. It can be said that the experimental program helped to improve reading comprehension for the students of the experimental group.

In addition to the tests, a survey questionnaire was also administered to the experimental group to find out how students evaluated the intervention program. The questionnaire consisted of five closed questions. The following table shows the results of the survey.

Table 2. *Students' evaluation of the intervention program*

No.	Questions	A (%)	B (%)	C (%)	D (%)
1.	*How much do you like the flipped reading lessons?* A. very much B. much C. not very much D. not at all	18.52	74.07	7.41	0
2.	*What do you think of the class atmosphere during flipped reading lessons* A. very interesting B. interesting C. boring D. very boring	22.22	77.78	0	0
3.	*What reading skills have you improved after the course? (more than one answer can be accepted)* A. Skimming for gist B. Scanning for details C. Understanding author's attitude D. Summarizing long texts	66.67	74.07	59.26	48.15
4.	*What other skills have you improved after the course (more than one answer can be accepted)* A. Being autonomous in learning reading skills B. Being more active in reading classes C. Being able to deepen a reading text D. Being able to find support materials from the Internet	96.30	88.89	85.19	100
5.	*What difficulties have you encountered in flipped reading lessons? (more than one answer can be accepted)* A. Too much pre-class preparation B. Working with un-prepared partners C. Not having an internet-connected computer D. Having too many new words in their chosen reading passages	74.07	37.04	29.63	22.22

The figures in the table show that most of the students liked flipped reading classes. 18.52% liked it very much, and up to 74.07% like it much. Only 7.41% did not like it very much. No student stated that they did not like flipped classes at all. Similarly, most of the students remarked flipped reading lessons are 'very interesting' (22.22%) and 'interesting' (77.78%). No student found them boring. In terms of reading skills, all the students stated that they improved such skills as skimming for gist (66.67%), scanning for details (74.07%), understanding author's attitude (59.26%) and summarizing long texts (48.15%). Moreover, the students also assumed that they improved other skills as well. 96.30% became autonomous in learning reading skills; 88.89% were more active in reading classes; 85.19 were able to deepen a reading text; and 100% were able to find support materials from the Internet. However, there still existed some difficulties for students. 74.07% complained there was too much pre-class preparation. 37.04% had to work with un-prepared partners one time or another. 29.63% did not have an internet-connected computer which made them difficult to follow the course. 22.22% had too many new words in their chosen reading passages. Class observation and informal interviews with the students of the experimental group also reinforced that flipped learning model really helped students to improve their reading comprehension in a novel way. They became more motivated, active and interested in the reading lessons.

4. Conclusion

Reading has always been considered an important language skill to be mastered for language learners. This study exploited flipped learning model to improve English majors' reading comprehension. Based on the theoretical framework of reading comprehension and flipped learning, steps for delivering a flipped reading lesson were put forward.

To find out whether this new model is effective in developing English majors' reading comprehension, an experimental research was conducted during 10 consecutive weeks of the reading course for third-year English majors at Hong Duc University. Two groups who were in their final reading course were chosen and randomly assigned as the control group and experimental group. While the control group was taught with traditional three steps of a reading lesson, the experimental one was provided with slipped reading lessons. A pretest and a posttest were administered to check the efficacy of the experimental program. Observation was also made in slipped classes throughout the semester. A survey questionnaire and informal interviews were conducted at the end of the course to find out students' reaction to slipped reading classes. The results from the tests, class observation, questionnaire and interviews demonstrated that slipped classes really helped students improve their reading comprehension, and made a positive change in the way students act in reading classes.

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RANDOM VARIATIONAL INEQUALITIES FOR SEMI H-MONOTONE MAPPINGS

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Received: 5 December 2017/ Accepted: 11 June 2019/ Published: June 2019

Abstract: *This paper is an extension of [2,4,6,7]. In this paper, one can solve some random variational inequalities for semi H-monotone and weakly semi H-monotone mappings.*

Keywords: *Random variational, semi H-momotone mapping.*

1. Notations and definitions

Let (Ω, Σ) be a measurable space, X and Z real Banach space, Z^* the dual of Z. We denote by $\langle z^*, z\rangle$ the dual pairing between $z^* \in Z^*, z \in Z$ and 2^X the set of the nonempty subsets of $X, cl(M)$ and $wcl(M)$, the respective closure and weak closure of $M \subset X$. Let $S_r = \{x \in X \mid \|x\| \le r, r > 0\}$, ∂S be the boundary of S. The notations "$\to$" and "$\rightharpoonup$" mean the strong and weak convergence respectively, $WK(D)$ is the set of weakly compact subsets of $D \subset X$. A mapping $T: \Omega \to 2^X$ is said to be measurable (weakly measurable) if for each closed measurable (weakly closed) subset $C \subset X$, the set $T^{-1}(C) = \{\omega \in \Omega \mid T(\omega) \cap C = \phi\} \in \Sigma$. A mapping $\xi: \Omega \to X$ is called measurable (weakly measurable) selector of a measurable (weakly measurable) mapping T if ξ is measurable and $\xi(\omega) \in T(\omega), \forall \omega \in \Omega$. A mapping $F: X \to X^*$ is said to be monotone if $\langle Fx - Fy, x - y\rangle \ge 0, \forall x, y \in X$. A mapping $K: X \to X$ is said to be J-monotone if $\langle J(x-y), Kx - Ky\rangle \ge 0, \forall x, y \in X$. Where mapping $J: X \to X^*$ is dual mapping, that is $\langle Jx, x\rangle = \|x\|^2, \|Jx\| = \|x\|, \forall x \in X$. A mapping $B: X \to Z$ is said to be weakly continuous if $\{x_n\} \subset X$, $x_n \rightharpoonup x$ then $Bx_n \rightharpoonup Bx$, completely continuous if $x_n \rightharpoonup x$ then $Bx_n \rightharpoonup Bx$, hemicontinuous if the mapping: $t \in [0,1], t \mapsto \langle B(tx + (1-t)y), z\rangle$ is continuous for all $x, y, z \in X$. A mapping $A: \Omega \times X \to Z$ is called a random mapping if for each fixed $x \in X$,

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the mapping $A(.,x):\Omega \to Z$ is measurable. A random mapping A is said to be continuous (weakly continuous, monotone,...) if for each $\omega \in \Omega$, the mapping $A(\omega,.):X \to Z$ has respective property. We use also $A(\omega)x$ for $A(\omega,x)$. We denote by $\mathcal{M}(\Omega,X)$ the set of measurable mappings $\xi:\Omega \to X$ such that $\sup\{\|\xi(\omega)\| \mid \omega \in \Omega\} < +\infty.$

Definition 1.1. (def. 2.1 in [6]). *Let X,Z be Banach spaces, Z^* the dual space of Z, $H:X \to Z^*$ a mapping satisfying $H(0)=0, Hx \neq 0, \forall x \neq 0$. A mapping $A:X \to Z$ is said to be H-monotone if $\langle H(x-y), Ax-Ay\rangle \geq 0, \forall x,y \in X$.*

Theorem 1.1. (theorem 2.3 in [6]) *Let X,Z be finite dimensional Banach spaces, $H:X \to Z^*$ a mapping satisfying $H(0)=0, H(x)\neq 0, \forall x \neq 0, A:X \to Z$ a continuous random mapping. Assume, moreover, there exists $r=constant>0$ such that for each $\omega \in \Omega, \langle Hx, A(\omega)x\rangle \geq 0, \forall x \in \partial S_r$. Then there exists $\xi \in \mathcal{M}(\Omega, S_r)$ such that $A(\omega)\xi(\omega)=0, \forall \omega \in \Omega$.*

2. Semi H-monotone mappings

Let X,Z be real Banach spaces. Consider the mappings $H:X \to Z^*, A:\Omega \times X \to Z.$ Let $\{X_n\},\{Z_n\}$ be inereasing sequences of finite dimensional subspaces of X and Z respectively, $\dim X_n = \dim Z_n$, and $P_n:X \to X_n, Q_n:Z \to Z_n, Q_n^*:Z^* \to Z_n^*$ linear projectors such that $P_n x \to x, Q_n z \to z$. Set $A_n = Q_n A|x_n, H_n = Q_n H|x_n$.

Definition 2.2. (def. 3.1 in [6]). *Let X,Z be Banach spaces, Z^* the dual space of Z, $H:X \to Z^*$ a mapping satisfying $H(0)=0, H(x)\neq 0, \forall x \neq 0.$ A mapping $A:X \to Z$ is said to be semi H-monotone if there exists a mapping $S:X \times X \to Z$ such that*

(i) $Ax = S(x,x), \ \forall x \in X$,

(ii) for each fixed $y \in Y$, the mapping S(., y) is H-monotone and hemicontinuous,

(iii) for each fixed $x \in X$, the mapping S(x, .) is completely continuous.

Theorem 2.2. *Let D be nonempty, convex, closed subset of a separable reflexive Banach space X,Z a separable reflexive Banach space, $H:X \to Z^*$ a weakly continuous mapping satisfying $H(0)=0, Hx \neq 0, \forall x \neq 0$ and for each $t>0, H(tx)=tHx$, $A:\Omega \times X \to Z$ a semi H-monotone random mapping. Supose, moreover, $Q_n^* Hx = Hx, \forall x \in X_n$ and for each finite dimensional subspace E of X, in $D_E = D \cap E$ there exists $\xi \in \mathcal{M}(\Omega, S)$ such that*

$$\left\langle H\left(\xi(\omega)-y\right), A(\omega)\xi(\omega)\right\rangle \le 0, \forall y \in D, \omega \in \Omega.$$

Proof. Let $D_n = D \cap X_n$. The sequence $\{D_n\}$ is increasing. Let us define mappings $H_n = Q_n H, A_n = Q_n A, H : X \to Z^*, A_n : \Omega \times D_n \to Z_n$. Obvionsly, $Q_n A$ is a continuous random mapping in D_n. For each $\omega \in \Omega$, we have

$$\left\langle Hx, Q_n A(\omega)x\right\rangle = \left\langle Q_n^* Hx, A(\omega)x\right\rangle = \left\langle Hx, A(\omega)x\right\rangle, \forall x \in \partial S_r.$$

The mapping $Q_n A$ satisfies all conditions of Theorem 1.1. So there exists $\xi \in \mathcal{M}(\Omega, S)$ such that $Q_n A(\omega)\xi(\omega) = 0, \forall \omega \in \Omega$. By the reflexivity of X the ball S is weakly compact. Let us consider mappings $B_n, B : \Omega \to WK(S)$ as follows:

$$B_n(\omega) = wcl\left\{\xi_n(\omega)\right\}, B(\omega) = \bigcap_{n=1}^{\infty} B_n(\omega).$$

As in the proof from [[9], p, 135] it is clear that B is weakly measurable and B has a measurable selector $\xi : \Omega \to S, \xi(\omega) \in B(\omega), \forall \omega \in \Omega$. Consequently, for each $\omega \in \Omega$, the sequence $\left\{\xi_n(\omega)\right\}$ has a subsequence denoted by $\left\{\xi_k(\omega)\right\}$ (for the simplicity of notations) weakly converging to $\xi(\omega)$. Moreover, for each $x \in S$ that is $x \in M_m$ for some m, and by the sequence D_n is increasing, obviously $x \in D_k, \forall k \ge m$. The semi H-monotonicity of the mapping A provides us a mapping $S : D \times D \to Z, A(\omega)x = S(\omega, x, x), \forall x \in D.$

Since the mapping $x \mapsto S(\omega, x, y)$ is H-monotone, we obtain

$$\left\langle H\left(\xi_k(\omega)-x\right), A(\omega)\xi_k(\omega) - S\left(\omega, x, \xi_k(\omega)\right)\right\rangle \ge 0 \tag{2.1}$$

But $\left\langle H\left(\xi_k(\omega)-x\right), A(\omega)\xi_k(\omega)\right\rangle = \left\langle H\left(\xi_k(\omega)-x\right), Q_k A(\omega)\xi_k(\omega)\right\rangle = 0$

It follows from inequality (2.1) that

$$\left\langle H\left(\xi_k(\omega)-x\right), S\left(\omega, x, \xi_k(\omega)\right)\right\rangle \le 0 \tag{2.2}$$

By $H\left(\xi_k(\omega)-x\right) \rightharpoonup H\left(\xi(\omega)-x\right)$ and $S\left(\omega, x, \xi_k(\omega)\right) \to S\left(\omega, x, \xi(\omega)\right)$ as $\xi_k(\omega) \to \xi(\omega)$ from inequality (2.2) we get

$$\left\langle H\left(\xi(\omega)-x\right), S\left(\omega, x, \xi(\omega)\right)\right\rangle \le 0 \tag{2.3}$$

The hemicontinuity of the mapping $S\left(\omega, ., \xi(\omega)\right)$ and inequality (2.3) yield

$$\left\langle H\left(\xi(\omega)-x\right), S\left(\omega, \xi(\omega), \xi(\omega)\right)\right\rangle \le 0$$

Or

$$\left\langle H\left(\xi(\omega)-y\right), A(\omega)\xi(\omega)\right\rangle \le 0, \forall y \in D, \omega \in \Omega.$$

Theorem 2.3. (H-monotone perturbation). *Let* D, X, Z, H, A *be as in Theorem . Let* $K:\Omega\times D\to Z$ *be a H-monotone, completely continuous random mapping. Assume, furthermore,* $Q_n^*Hx = Hx, \forall x\in X_n$ *and for each finite dimensional subspace* E *of* X *, in* $D_E = D\cap E$*, there exists a ball* S *such that*

$$\left\langle Hx, QA(\omega)x\right\rangle \geq 0 \text{ and } \left\langle Hx, K(\omega)x\right\rangle \geq 0, \forall y\in D, \omega\in\Omega.$$

Then there exists $\xi\in\mathcal{M}(\Omega, S)$ *such that*

$$\left\langle H\left(\xi(\omega)-y\right), A(\omega)\xi(\omega)+K(\omega)\xi(\omega)\right\rangle \leq 0, \forall y\in D, \omega\in\Omega.$$

Proof. Let us use the notations $D_n, Q_n^*H, Q_nA, Q_nK:\Omega\times D_n\to Z_n$ as in the proof of Theorem 2.2. The mapping Q_nA, Q_nK are continuous in D_n. So they satisfy all conditions in Theorem 1.1. Consequently there exists $\xi\in\mathcal{M}(\Omega, S)$ such that $Q_nA(\omega)\xi(\omega)=0, Q_nK(\omega)\xi(\omega)=0$. Let us use the mappings B_n, B in the proof of Theorem 2.2. It is clear that B is weakly measurable and B possesses a measurable selector ξ. Hence the sequence $\{\xi_n(\omega)\}$ weakly converging to $\{\xi(\omega)\}$. The semi H-monotonicity of K yield $\left\langle H\left(\xi_k(\omega)-x\right), S(\omega, x,)\xi_k(\omega)+K(\omega)\xi_k(\omega)\right\rangle \leq 0$

whence $$\left\langle H\left(\xi(\omega)-x\right), S(\omega, x,)\xi(\omega)+K(\omega)\xi(\omega)\right\rangle \leq 0 \tag{2.4}$$

or $$\left\langle H\left(\xi(\omega)-y\right), A(\omega)\xi(\omega)+K(\omega)\xi(\omega)\right\rangle \leq 0, \forall y\in D, \omega\in\Omega.$$

3. Weakly semi H-monotone mappings

Definition 3.3. (def. 4.1 in [6]). *Let* X, Z, Z^* *be as in Definition 1.1. A mapping* $A: X\to Z$ *is said to be weakly semi H-monotone if there exists a mapping* $R: X\times X\to Z$ *$R: X\times X\rightarrow Z$ such that*

(i) $Ax = R(x,x), \forall x\in X$,

(ii) for each fixed $y\in X$ *, the mapping* $R(.,y)$ *is H-monotone and hemicontinuous.*

(iii) for each fixed $x\in X$ *, the mapping* $R(x,.)$ *is weakly continuous.*

Obvionsly the semi H-monotonicity implies the weak semi H-monotonicity and in finitely dimensional space in which those concepts coinside.

Theorem 3.4. *Let* D, X, Z *be as in Theorem 2.2,* $H: X\to Z^*$ *be a completely continuous mapping*

$H(0)=0, Hx\neq 0, \forall x\neq 0$ *and for each* $t>0, H(tx)=tHx, A:\Omega\times D\to Z$ *a weakly semi H-monotone random mapping.*

Suppose, furthermore, $Q_n^* Hx = Hx, \forall x \in X_n$ *and for each finite dimensional subspaces* E *of* X *, in* $D_E = D \cap E$ *there exists a ball* S *such that* $\langle Hx, A(\omega)x \rangle \geq 0, \forall x \in \partial S$*. Then there exists* $\xi \in \mathcal{M}(\Omega, S)$ *such that* $\langle H(\xi(\omega) - y), A(\omega)\xi(\omega) \rangle \leq 0, \forall y \in D, \omega \in \Omega$..

Proof. Let us use the notations D_n, A_n, H_n in the proof Theorem 2.2. The mapping $Q_n A$ is continuous in D_n. Moreover $\langle H_n x, Q_n A(\omega)x \rangle = \langle Hx, A(\omega)x \rangle \geq 0, \forall x \in \partial D$.

Hence the random mapping $Q_n A$ satisfies all conditions of Theorem 1.1. So there exists $\xi \in \mathcal{M}(\Omega, S)$ such that $Q_n^* A(\omega)\xi_n(\omega) = 0$.

By using the mapping $\omega \mapsto B(\omega) = \bigcap_{n=1}^{\infty} B_n(\omega)$ as in the proof of Theorem 2.2, it is clear that B has a measurable selector $\xi, \xi(\omega) \in B(\omega), \forall \omega \in \Omega$. Consequently for each $\omega \in \Omega$, the sequence $\{\xi_n(\omega)\}$ provides us a subsequence, say $\{\xi_n(\omega)\}$ weakly converging to $\{\xi(\omega)\}$ and for each $x \in D$, we see $x \in D_k, \forall k \geq m$ for some m. By the H-monotonicity of the mapping $R(\omega, ., y)$, where $R(\omega, x, x) = A(\omega)x$, we obtain $\langle H(\xi_k(\omega) - x), A(\omega)\xi_k(\omega) - R(\omega, x, \xi_k(\omega)) \rangle \geq 0,$

which implies $\langle H(\xi_k(\omega) - x), R(\omega, x, \xi_k(\omega)) \rangle \leq 0, \qquad (3.5)$

But $H(\xi_k(\omega) - x) \to H(\xi(\omega) - x)$, $R(\omega, x, \xi_k(\omega)) \rightharpoonup R(\omega, x, \xi(\omega))$ as $\xi_k(\omega) \rightharpoonup \xi(\omega)$. Therefore from inequality (3.5) it follows that

$$\langle H(\xi(\omega) - x), R(\omega, x, \xi(\omega)) \rangle \leq 0,$$

The hemicontinuity of $R(\omega, ., \xi(\omega))$ and inequality (3.6) yield

$$\langle H(\xi(\omega) - y), A(\omega)\xi(\omega) \rangle \leq 0, \forall y \in D, \omega \in \Omega. \qquad (3.6)$$

It is not difficult to prove.

Theorem 3.5. *Let* D, X, Z, H, A *be as in Theorem 3.4,* $K : \Omega \times D \to Z$ *be a H-monotone, weakly continuous random mapping. Assume, moreover,* $Q_n^* Hx = Hx, \forall x \in X_n$ *and for each finite dimensional subspace* E *of* X *, in* $D_E = D \cap E$*, there exists a ball* S *such that* $\langle Hx, A(\omega)x \rangle \geq 0, \langle Hx, K(\omega)x \rangle \geq 0, \forall x \in \partial S$ *. Then there exists* $\xi \in \mathcal{M}(\Omega, S)$ *such that*

$$\langle H(\xi(\omega) - y), A(\omega)\xi(\omega) + K(\omega)\xi(\omega) \rangle \leq 0, \forall y \in D, \omega \in \Omega.$$

4. Conclusion

The theorems 3.4 and 3.5 solve some random variational inequalities for semi H-monotone and weakly semi H-monotone mappings. These are good results in solving random variational inequalities for semi H-monotone and weakly semi H-monotone mappings.

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USING REMOTE SENSING AND GIS TO ANALYZE LAND COVER/LAND USE CHANGE IN QUANG XUONG DISTRICT, THANH HOA PROVINCE, VIETNAM

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Received: 4 June 2018/ Accepted: 11 June 2019/ Published: June 2019

Abstract: *Identification of land use change in different periods of time has become a central key to monitoring of land resources. It is relatively important for effective land management to protect the land resources, especially the land used for agricultural production from overuse and environmental changes. The sprawl of inhabitant areas, development of rural infrastructures, and industrialization are responsible for serious losses of agricultural land. In this study, remote sensing techniques were applied to studying the trends of land cover change in the Quang Xuong District in a period of about 24 years from 1989 to 2013. ArcGIS software was adopted to develop the land cover and the change of land use maps from 1989 to 2013. Two satellite images with moderate resolution were collected from USGS Earth Explorer website, Landsat 5 TM for 1989 and Landsat 8 OLI & TIRS for 2013. After image geo-processing, the images were classified into six land cover categories by applying supervised classification method (Maximum Likelihood). The six main obtained land cover types were built-up areas, agricultural land, forest land, water surface area, salty land, and unused land. The overall accuracies of land cover maps for 1989 and 2013 were 94.08% and 92.91%, respectively. The results of change detection analysis indicate that the cultivated, water surface and unused lands decreased by 22%, 17%, and 91%, respectively. In other side, the built-up and salty land increased by 78%, 58%, respectively and forest land increased from 52.69 ha in 1989 to 395.76 ha in 2013.*

Keywords: *Remote sensing, Landsat, Quang Xuong District, Change detection, Land cover/Land use.*

1. Introduction

Presently, remote sensing (RS) is used as a powerful tool that can be applied to handle the problem of thematic maps which have to be updated. It has capabilities to map and extract information of the earth resources for different purposes. RS and GIS have abilities to create update solutions, build and analyze data efficiently [14]. According to Thom and Que (2014), RS and GIS are leading to dramatic changes in the management of natural resources because of their outstanding advantages such as shortening time, increasing accuracy, logic, and the current state of the map information [15].

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One of the most important applications of RS is land cover mapping [3]. According to Casady and Palm (2002) RS for agriculture can be defined as “observing or a field crop without touching it”. It integrates new technologies that can offer increasingly efficient, complete, precise and timely information [4].

RS has been used in studies on vegetation for many years with various perspectives such as building the map of forest fires, vegetation cover or detecting changes in vegetation through different periods [7], [12]. Townshend et al., (1991) used RS to calculate the changes in the vegetative cover of the land surface on a global scale [16]. RS has been widely used in natural research for mapping vegetation since it can quickly determine the data, distribution, and change of vegetation for large areas. In addition, it provides the possibility of inferring results of mapping to regional extent, even in large inaccessible areas [11]. Using RS to create a picture or map is a quick approach for calculating the extent of an essential crop characteristics or a field that has the same characteristics [4].

One of the most influential factors causing ecological systems and climate change is land cover change [18]. It reflects human activities and physical environments on Earth. Information about land use and land cover is needed for water-resources inventories, flood control, water supply planning, and waste water treatment [1]. However, knowledge of land cover and its dynamics is particularly limited by the paucity of accurate land cover data [9]. Primary causes of changes to land use are commonly urbanization and new residential settlements, which has impacts on local communities’ environmental, social and economic sutainability [20].

In this study, maximum likelihood supervised classification and post-classification change detection techniques were used to find out land cover changes over the period of 1989 - 2013 in Quang Xuong District. Land cover monitoring of the research site over-time demanded a specific dataset of Landsat imageries in order to meet different local land use changes. This was one of the first important tasks in the project of land use planning and land evaluation. Moreover, monitoring of land cover also provided precious information for land users, decision makers, and land planners to make reasonable development strategies of land use in the short-term as well as in the long-term.

2. Study area

Quang Xuong is one of a coastal district of Thanh Hoa Province and is located in the tropical and temperate zone. It’s geographical location is at $19^0$34’ - $19^0$47’N latitude and $105^0$46’ - $105^0$53’E (Figure 1). The topography of Quang Xuong District is saddleback and relatively flat, which runs from the north to the south. The average height above sea level is from 3 to 5 meters. Similar to the climate of the entire province, this district is characterizes by strong monsoon influence, a considerable amount of sunny days, and with a high rate of rainfall and humidity. The weather of the district is divided into four distinct seasons: spring, summer, autumn and winter. It is hot and humid weather by influence of the south-westerly dry wind in the summer; dry and little rain, occasional appearance of frost in the winter. The total temperature is about 8300 - 8400^0C per year. The annual average precipitation ranges from 1600mm to 2000mm and is irregularly distributed. The humidity is rather high. The average account is over 80% in most of the months and is rarely under 60%.

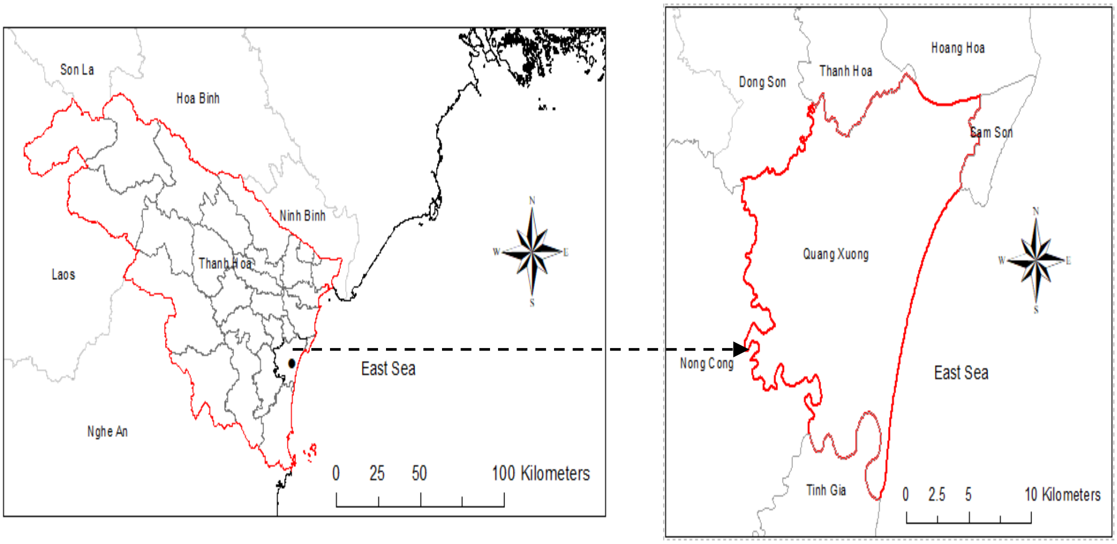

Figure 1*. Location of the study area*

3. Materials and Methodology

3.1. Satellite Data

Landsat 5 TM acquired on September of 1989 and Landsat 8 OLI and TIRS acquired on September of 2013 (Table 1) were used for classification and land cover change detection from 1989 to 2013. The data were selected for this study as it has wide spectral coverage and availability of a high resolution band for enhancing spatial resolution and features. These satellite images were acquired for relatively cloud free (maximum 10%) in both period of time for visual interpretation and on screen digitizing.

Table 1. *Characteristics of Landsat 5 TM, Landsat 8 OLI and TIRS data*

Image	Resolution	Path	Row	Date of pass
Landsat TM	30.0m (band 1-5 & 7)	127	046	September 11, 1989
Landsat OLI and TIRS	30.0m (band 1-7 & 9)	126	046	September 22, 2013

3.2. Methodology

The principle of classification based on the land cover and land use classification system developed by A. Anderson et al., (1976) was applied first. Supervised classification approach was independently used for classification stage for each image to generate the thematic map of land cover; afterward change detection technique was also applied to examine how land cover change from 1989 to 2013 in Quang Xuong District by comparing independently classified images. Six separable land cover types have been identified in this research including water surface, built-up, agricultural land, forest land, salty land, and unused land (Table 2).

Table 2. *Land cover classification scheme*

No.	Classes	Description
1	Built-up area	Area covered by residential, commercial, industrial, public infrastructure and services buildings, transportation, roads, mixed urban, and other urban.
2	Agriculture land	Characterized by agricultural area, crop fields, fallow lands, vegetable lands and regularly planted crops.
3	Water surface	All area of open water with 95% covers of water, including rivers, streams, lakes, ponds and reservoirs.
4	Forest	Area covered by forest with relatively darker green colors.
5	Salty land	Area used for salt production.
6	Unused land	Sandy, rock mountains and other disused areas.

3.2.1. Image pre-processing

In this study, Landsat TM of 1989 and Landsat OLI & TIRS of 2013 were rectified to UTM zone 48, WGS 84. The geometric correction of the images was performed using topography of Quang Xuong district with the help of Ground control points (GCPs). As to prevent possible changes to the original pixel values of the image data, neighbor resampling method was applied. Therefore, both images of 1989 and 2013 were geometrically corrected by using 35 control points. The root mean square errors (RMSE) for Landsat TM of 1989 and Landsat OLI of 2013 were 0.020 pixels and 0.017 pixels, respectively. The next stage was clipping the images to focus on the processing of the study area.

3.2.2. Selection of training samples

The training samples were selected based on the basis of the unsupervised classified image and the current land use map of 2012. These training samples were selected from all cover land founding in the study area with the average of 26 training areas for each land cover type of 1989, 30 training areas for each land cover type of 2013, and a minimum average of 12 pixels for each training sample of both images. Besides, the statistical analyses were computed based on Jeffrey-Matusita distance [19]. The number of land use/land cover classes were defined based on field work and available land use statistics for the study area, and the defined classes for image classification were Built-up, Agriculture land, Water surface, Forest land, Salty land, and Unused land area.

3.2.3. Image classification

In the next step, the supervised classification is applied for the classification process. It is performed with the maximum-likelihood algorithm, where the training samples are homogeneous reflectance of certain areas. This approach demonstrates that data is best collected from remote areas if each class contains some Gaussian distribution [2]. In this stage, the maximum likelihood classifier was conducted, since it could obtain some reliable results. Contrarily, parallelepiped classifier would bring problem when overlapping and minimum distance classifier is insensitive to the discrepancy in each class. Finally, the classified images were smoothed by using a 3 × 3 majority filter to reduce the number of misclassified pixel in the land use/land cover maps [8].

3.2.4. Accuracy assessment

The number of the reference pixels is a key component in computing the precision of classification. According to Congalton (1991), it needed more than 250 reference pixels to determine the means of a class to within plus or minus five percent [6]. In this research, a standard method suggested by Congalton (1991) was used to assess the overall accuracy, producer's and user's accuracy. After performing the image classification, the results of the accuracy assessment were presented in the confusion matrix by using quantitative analysis. Furthermore, a discrete multivariate approach of Kappa analysis is also used in accuracy assessment from the confusion matrix [13]. It is known as a Khat statistic approach to measure the agreement or accuracy [5]. The Kappa statistic illustrates the agreement between the classified land use and the observed land use. Unlike the overall, producer's and user's accuracies, in general, Kappa analysis can take the chance allocation of class labels into consideration by using the main diagonal, columns, matrix rows, and error matrix [17]. The Kappa statistic is calculated as:

$$Kappa(K) = \frac{N\sum_{i=1}^{r} x_{ii} - \sum_{i=1}^{r}(x_i \times x_{+i})}{N^2 - \sum_{i=1}^{r}(x_i \times x_{+i})}$$

Where r is the number of rows in the matrix, X_{ii} is the number of observations in row i and column i, $x_i = x_{+i}$ is marginal totals for row i and column i respectively, and N is the total number of pixels.

3.2.5. Post classification processing

The land use/land cover classification was generated by two Landsat TM and Landsat OLI & TIRS images acquired in September of 1989 and 2013. After classification, detection of land cover changes was achieved by overlaying and post-classification comparison of the land cover/land use maps of the different time periods. This step gave not only the size and distribution of changed areas, but also the percentages of other land cover classes that share in the change of each land cover class individually. For the maximum quality of spectral data from classification process, the original resolution of the satellite images was used to determine the quantity of the conversions [10]. The map of the change was accompanied by the respective cross tabulation matrices showing the change pathways.

4. Results and discussion

4.1. Land cover/Land use status in 1989 and 2013

The land use/land cover classification was examined based on the results from the interpretation of two Landsat images acquired in September of 1989 and 2013 (Table 2 and Figure 2). Afterwards, the results of classification were exported to ArcGIS for further processing.

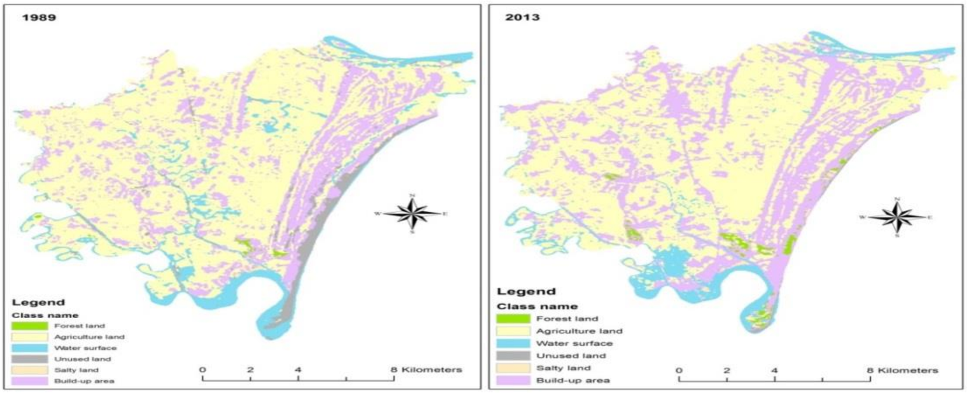

***Figure 2**. Supervised Maximum likelihood classification of 1989 and 2013*

Table 2 shows that approximately 63% and 49.17% of the total area was for agricultural uses in 1989 and 2013. The built-up area covered approximately 22.7% and 40.32% of the total geographical area of Quang Xuong District in 1989 and 2013, respectively. The water surface covered about 9.3% and 7.72% of the total area of the region in 1989 and 2103, respectively. About 0.3% and 0.42% area was under salty practices in 1989 and 2013, respectively. There was about 0.2% and 1.97% of the total study area under the forest cover in 1989 and 2013, respectively. The unused area covered about 4.5% and 0.40% of the total natural area in 1989 and 2013. The spatial pattern reveals that the study area is flat and more than a half of the natural area was used for agricultural practices in 1989 and nearly half of the area used for agricultural activities in 2013. However, the natural area for agricultural productivity is decreasing due to the expansion area for inhabitants as well as for rural infrastructure development.

***Table 2.** Land cover/land cover classification in 1989 and 2013*

No.	**Class**	**1989**		**2013**		**Change**
		Area (ha)	%	Area (ha)	%	
1	Water surface	2122.29	9.3	1759.48	7.72	-362.81
2	Salty land	60.51	0.3	95.47	0.42	35.16
3	Built-up area	5172.48	22.7	9185.44	40.32	4012.96
4	Agriculture land	14362.12	63.0	11200.30	49.17	-3161.82
5	Forest land	52.69	0.2	448.45	1.97	395.76
6	Unused land	1010.25	4.5	91.00	0.40	-919.25

4.2. Accuracy assessment

Accuracy assessment was examined for image classification of 1989 and 2013. A stratified random sampling design was adopted in the accuracy assessment. For the land use/land cover classification of 1989, a total of 591 pixels were randomly selected. The results indicated that an overall accuracy is of 94.08% and a Kappa index of agreement is of 0.91 (Table 3). In examining the producer's accuracy, all classes are over 85%, except salty land which was 77.78%. In examining of the user's accuracy, all classes are over 90%, except forest land which was 87.50%.

Table 3. *Accuracy assessment of Landsat 5 TM of 1989*

Classified data	Reference data 1989							
	Agricultural land	**Build-up-land**	**Water surface**	**Salty land**	**Unused land**	**Forest land**	**Row total**	**User's accuracy (%)**
Agricultural land	**289**	6	5	1	2	1	304	95.07
Build-up area	6	**112**	0	0	4	0	122	91.80
Water surface	3	0	**87**	0	0	0	90	96.67
Salty land	0	0	0	**7**	0	0	7	100.00
Unused land	0	1	3	1	**47**	0	52	90.38
Forest land	1	1	0	0	0	**14**	16	87.50
Column total	299	120	95	9	53	15	**591**	
Producer's accuracy (%)	96.66	93.33	91.58	77.78	88.63	93.33		
Overall accuracy = 94.08% Kappa index = 0.91								

For the land use/land cover classification of 2013, a total of 494 pixels were selected. The results presented that an overall accuracy is of 92.91% and a Kappa index of agreement is of 0.896 (Table 4). In term of the producer's accuracy, all classes are over 90%, except salty land class which made up 66.67%. In terms of the user's accuracy four classes exhibit over 90% with the exception of salty and unused land classes, which are 54.55% and 68.75%, respectively. The salty and unused land classes show clear confusion because of similar reflection value of them.

Table 4. *Accuracy assessment of Landsat 8 of 2013*

Classified data	Reference data 2013							
	Agricultural land	**Build-up land**	**Water surface**	**Salty land**	**Unused land**	**Forest land**	**Row total**	**User's accuracy (%)**
Agricultural land	**157**	1	1	0	0	11	170	92.35
Build-up area	2	**198**	0	2	1	1	204	97.06
Water surface	3	2	**63**	0	0	0	68	92.65
Salty land	0	0	5	**6**	0	0	11	54.55
Unused land	0	3	1	1	**11**	0	16	68.75
Forest land	0	1	0	0	0	**24**	25	96.00
Column total	162	205	70	9	12	36	**494**	
Producer's accuracy (%)	96.91	92.09	90.00	66.67	91.67	92.31		
Overall accuracy = 92.91%								
Kappa index = 0.896								

5.3. Land cover/Land use change detection

The surface distribution (in ha) of the proportion of each land cover/land use class in the different time from 1989 to 2013 is as presented in Table 1. All the land cover types have been changed from 1989 to 2013, the largest change namely build-up area, cultivated, unused, and forest lands. Table 1 shows that about 3,161.82ha, 919.25ha and 362.81ha decrease is observed in agricultural areas, unused land, and water surface areas. Meanwhile there is an increase of 4,012.96ha in built-up area, 395.76ha in forest, and 35.16ha in salty lands between 1989 and 2013. The detail dynamics of the land use/land cover changes in the study area between 1989 and 2013 is shown in Table 5. The table is a cross tabulation matrix of the land use/land cover change, displaying the conversion from each class to another class. For instance, from 1989 to 2013, 10130.41ha agricultural area production remained stable, 1069.89ha of new cultivated land are mostly generated at the expense of water surface, unused land and build-up area. Contrarily, 4231.71ha of agricultural land are lost to built-up areas (3323.18ha), water surface (757.06ha), forest (126.23ha), unused land (15.62ha), and salty land (10.62ha). The land cover categories of forest and build-up areas are expanded the most over other types of land use, with 395.76ha and 4012.96ha, respectively, mostly from cultivated, water surface, unused areas. During this period of 24 years, the area of agricultural, unused lands and water surface are the greatest reduction in area, with 4231.71ha, 1156.11ha and 988.11ha, respectively.

Table 5. *The change of land cover/land use from 1989 to 2013 in ha*

1989 - 2013	Forest land	Agriculture land	Water surface	Unused land	Salty land	Built-up area	Total	Expansion
Forest land	**25.43**	126.23	15.34	233.56	7.2	40.69	448.45	423.02
Agriculture land	13.95	**10130.41**	473.61	98.68	0.1	483.55	11200.3	1069.89
Watersurface	0.13	756.06	**966.18**	26.89	0.41	9.81	1759.48	793.3
Unused land	0	15.62	41.28	**22.14**	1.98	9.98	91	68.86
Salty-land	0	10.62	41.22	29.43	**13.86**	0.54	95.67	81.81
Built-up area	13.18	3323.18	584.66	499.55	36.96	**4727.91**	9185.44	4607.53
Total	52.69	14362.12	2122.29	1010.25	60.51	5173.66	22781.34	
Reduc-tion	27.26	4231.71	1156.11	988.11	46.65	594.57		

5. Conclusion

Not any studies have been applied remote sensing and satellite images to analyze land use change in Quang Xuong District before. At the time, the land cover distribution of the proportion from 1989 to 2013 in this research was the very first study of land cover change detection by applying remote sensing techniques and Landsat images in this district. The results of image classification in this study illustrate that the Maximum Likelihood supervised method is a useful tool for classifying and mapping broad categories of land cover/land use. In addition, change detection statistics is a helpful approach for determining the change of land cover/land use in different period of times. These results clearly suggest that satellite images of Landsat could be used to identify, classify and compare the change of land cover types in Quang Xuong District in particular and Thanh Hoa Province in general.

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NON-AUTONOMOUS STOCHASTIC EVOLUTION EQUATIONS, INERTIAL MANIFOLDS AND CHAFEE-INFANTE MODELS

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Received: 28 January 2019/ Accepted: 11 June 2019/ Published: June 2019

Abstract: *Consider a stochastic evolution equation containing Stratonovich-multiplicative white noise of the form* $\frac{du}{dt} + Au = f(t,u) + u^\circ \dot{W}$ *where the partial differential operator* A *is positive definite, self-adjoint with a discrete spectrum; and the nonlinear part* f *satisfies the* φ *-Lipschitz condition with* φ *belonging to an admissible function space. We prove the existence of a (stochastic) inertial manifold for the solutions to the above equation. Our method relies on the Lyapunov-Perron equation in a combination with the admissibility of function spaces. An application to the non-autonomous Chafee - Infante equations is given to illustrate our results.*

Keywords: *Stochastic inertial manifold;* φ *- Lipschitz; Admissibility, Lyapunov - Perron equation, nonautonomous Chafee - Infante equations.*

1. Introduction

In the present paper, we study the existence of an inertial manifold for a class of stochastic partial differential equations (SPDE) in which the nonlinear part is assumed to be φ -Lipschitz. Concretely, we will prove the existence of an inertial manifold for the following stochastic evolution equation driven by linear multiplicative white noise in the sense of Stratonovich

$$\frac{du}{dt} + Au = f(t,u) + u^\circ \dot{W} \tag{1.1}$$

where A is a positive definite, self-adjoint, closed linear operator with a discrete spectrum; f is φ - Lipschitz (see Definition 2.3); and $u^\circ \dot{W}$ is the noise.

There are two main difficulties when we transfer to the case of SPDE with φ - Lipschitz nonlinear term f : Firstly, since the nonlinear operator f is φ -Lipschitz, the existence and uniqueness theorem for solutions to (1.1) is not available. Secondly, the appearance of the white noise changes the formula of mild solutions for SPDE, and therefore changes the representation of Lyapunov-Perron equation used in the construction of the inertial manifold.

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To overcome such difficulties, we reformulate the definition of inertial manifolds suchthat it contains the existence and uniqueness theorem as a property of the manifold (see Definition 2.5 below). Furthermore, we construct the structure of the mild solutions to (1.1) using the white noise in such a way that it allows to combine the exponential estimates of the linear part of Eq. (1.1) with the existence and uniqueness of its bounded solutions (in negative direction) in the case of φ-Lipschitz nonlinear forcing terms. Consequently, we obtain the existence of an inertial manifold for semi-linear SPDE with φ -Lipschitz nonlinear term and general spectral gap conditions.

Our main result is contained in Theorem 2.8 which extends the results in [12] to the case of semilinear SPDE. Finally, we apply the obtained result to the nonautonomous Chafee - Infante equations (see Section 4).

2. Inertial Manifolds

Throughout this paper we assume that A is a positive definite, self-adjoint, closed and linear operator on a separable Hilbert space X with a discrete spectrum, say

$$0<\lambda_1\le\lambda_2\le\cdots,\quad \text{each with finite multiplicity and} \quad \lim_{k\to\infty}\lambda_k=\infty.$$

Let $\{e_k\}_{k=1}^{\infty}$ be the orthonormal basis in X consisted of the corresponding eigenfunctions of A (i.e., $Ae_k=\lambda_k e_k$).

Let then λ_N and λ_{N+1} be two successive and different eigenvalues with $\lambda_N<\lambda_{N+1}$, let further P be the orthogonal projection onto the first N eigenvectors of the operator A. Denote by $(e^{-tA})_{t\ge 0}$ the semigroup generated by $-A$.

Since ImP is finite dimension, we have that the restriction $(e^{-tA}P)_{t\ge 0}$ of the semigroup $(e^{-tA})_{t\ge 0}$ to ImP can be extended to the whole line $\mathbb{R}$.

For $0\le\theta<1/2$ we then recall the following dichotomy estimates (see [22]):

$$\|e^{-tA}P\|\le Me^{\lambda_N|t|},\quad t\in\mathbb{R} \text{ for some constant } M\ge 1,$$

$$\|A^{\theta}e^{-tA}P\|\le\lambda_N^{\theta}Me^{\lambda_N|t|},\quad t\in\mathbb{R},$$

$$\|e^{-tA}(I-P)\|\le Me^{-\lambda_{N+1}t},\quad t\ge 0,$$

and (2.1)

$$\|A^{\theta}e^{-tA}(I-P)\|\le M\left[\left(\frac{\theta}{t}\right)^{\theta}+\lambda_{N+1}^{\theta}\right]e^{-\lambda_{N+1}t},\quad t>0,\theta>0.$$

Next, we recall some notions on function spaces and refer to Massera and Schaffer [19], Rabiger and Schnaubelt [20], and Huy [11] for concrete applications.

Denote by $\mathcal{B}$ the Borel algebra and by λ the Lebesgue measure on $\mathbb{R}$.

The space $L_{1,loc}(\mathbb{R})$ of real-valued locally integrable functions on $\mathbb{R}$ (modulo λ -nullfunctions) becomes a Frechet space for the seminorms $p_n(f) := \int_{J_n} |f(t)|\, dt$, where $J_n = [n, n+1]$ for each $n \in \mathbb{N}$.

We can now define Banach function spaces as follows

Definition 2.1. [12] A vector space E of real-valued Borel-measurable functions on $\mathbb{R}$ (modulo λ -nullfunctions) is called a *Banach function space* (over $(\mathbb{R}, \mathcal{B}, \lambda)$) if

(1) E is Banach lattice with respect to a norm $\|\cdot\|_E$, i.e., $(E, \|\cdot\|_E)$ is a Banach space, and if $\varphi \in E$ and ψ is a real-valued Borel-measurable function such that $|\psi(\cdot)| \le |\varphi(\cdot)|$, λ -a.e., then $\psi \in E$ and $\|\psi\|_E \le \|\varphi\|_E$,

(2) the characteristic functions χ_A belongs to E for all $A \in \mathcal{B}$ of finite measure,

$$\sup_{t\in\mathbb{R}} \|\chi_{[t,t+1]}\|_E < \infty \quad \text{and} \quad \inf_{t\in\mathbb{R}} \|\chi_{[t,t+1]}\|_E > 0,$$

(3) E $L_{1,loc}(\mathbb{R})$, i.e., for each seminorm p_n of $L_{1,loc}(\mathbb{R})$ there exists a number $\beta_{p_n} > 0$ such that $p_n(f) \le \beta_{p_n} \|f\|_E$ for all $f \in E$.

We remark that condition (3) in the above definition means that for each compact interval $J \subset \mathbb{R}$ there exists a number $\beta_J \ge 0 \ne$ such that $\int_J |f(t|)|\, dt \le \beta_J \|f\|_E$ for all $f \in E$.

Definition 2.2. [12] The Banach function space E is called *admissible* if

(1) there is a constant $M \ge 1$ such that for every compact interval $[a,b] \subset \mathbb{R}$ we have

$$\int_a^b |\varphi(t)|\, dt \le \frac{M(b-a)}{\|\chi_{[a,b]}\|_E} \|\varphi\|_E \quad \text{for all } \varphi \in E, \tag{2.2}$$

(2) for $\varphi \in E$ the function $\Lambda_1\varphi$ defined by $\Lambda_1(t) := \int_{t-1}^{t} \varphi(\tau)d\tau$ belongs to E.

(3) E is T_τ^+ - invariant and T_τ^- - invariant, where T_τ^+ and T_τ^- are defined for $\tau \in \mathbb{R}_+$ by

$$T_\tau^+\varphi(t) := \varphi(t-\tau) \quad \text{for } t \in \mathbb{R}, \tag{2.3}$$

$$T_\tau^-\varphi(t) := \varphi(t+\tau) \quad \text{for } t \in \mathbb{R}.$$

Moreover, there are N_1, N_2 such that $\|T_\tau^+\| \le N_1, \|T_\tau^-\| \le N_2$ for all $\tau \in \mathbb{R}_+$.

Next, we introduce the notion of φ -Lipschitz function in the following definition.

Definition 2.3. For $\theta \in [0,1/2)$ put $X_\theta := D(A^\theta)$. Let E be an admissible Banach function space on $\mathbb{R}$ and φ be a positive function belonging to E. A function $f : \mathbb{R} \times X_\theta \to X$ is said to be φ - Lipschitz if f satisfies

$\|f(t,x)\| \le \varphi(t)\left(1+\left\|A^\theta x\right\|\right)$ for a.e. $t \in \mathbb{R}$ and all $x \in X_\theta$;

$\left\|f(t,x_1) - f(t,x_2)\right\| \le \varphi(t)\left\|A^\theta (x_1 - x_2)\right\|$ for a.e. $t \in \mathbb{R}$ and all $x_1, x_2 \in X_\theta$.

We can define the Green function as follows

$$G(t,s) = \begin{cases} e^{-(t-s)A}(I-P) & \text{for } t > s, \\ -e^{-(t-s)A}P & \text{for } t \le s. \end{cases} \tag{2.4}$$

Then, one can see that $G(t,s)$ maps X into X_θ. Also, by the dichotomy estimates and for $\gamma = (\lambda_N + \lambda_{N+1})/2$ we have

$$\left\|e^{\gamma(t-s)} A^\theta G(t,s)\right\| \le K(t,s) e^{-\alpha|t-s|} \text{ for all } t,s \in \mathbb{R} \tag{2.5}$$

where $\alpha = (\lambda_{N+1} - \lambda_N)/2$ and $K(t,s) = \begin{cases} M\left(\left(\dfrac{\theta}{t-s}\right)^\theta + \lambda_{N+1}^\theta\right) & \text{if } t > s \\ M\lambda_N^\theta & \text{if } t \le s \end{cases}$.

We then recall the definition of metric dynamical systems (MDS) associated with the Wiener process which will be used throughout this paper. For details on these notions we refer the reader to [1,4,9,17,18,21].

Definition 2.4. [1] A family of mappings $\{\theta_t\}_{t\in\mathbb{R}}$ on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ is called a *metric dynamical system* (MDS) if the following conditions are satisfied

(i) $\theta_0 = Id_\Omega$, and $\theta_{t+s} = \theta_t \circ \theta_s$ for all $t,s \in \mathbb{R}$;

(ii) The map $(t,\omega) \mapsto \theta_t\omega$ is $(\mathcal{B} \otimes \mathcal{F}; \mathcal{F})$ - measurable;

(iii) $\mathbb{P}$ is invariant respect to θ_t for all $t \in \mathbb{R}$;

In this paper, we deal with the MDS induced by the Wiener process. Precisely, let W_t be a two-sided Wiener process with trajectories in the space $C_0(\mathbb{R},\mathbb{R})$ of real continuous functions defined on $\mathbb{R}$, taking zero value at $t = 0$; $\mathcal{F}$ is the Borel σ - algebra associated with the Wiener process; $\mathbb{P}$ is the classical Wiener measure on Ω and for each $t \in \mathbb{R}$ the mapping $\theta_t : (\Omega, \mathcal{F}, \mathbb{P}) \to (\Omega, \mathcal{F}, \mathbb{P})$ is defined by

$$\theta_t\omega(\cdot) = \omega(\cdot + t) - \omega(t). \tag{2.6}$$

Moreover, we will consider a subset $\Omega \subset C_0(\mathbb{R},\mathbb{R})$, which is invariant under $\{\theta_t\}_{t\in\mathbb{R}}$, i.e., $\theta_t\Omega = \Omega$ for $t \in \mathbb{R}$.

Now, we make precisely the notion of a stochastic inertial manifold, and then prove its existence for solutions to SPDE (1.1). To do this, we first rewrite equation (1.1) in a more convenient form. To this purpose, let $z(\cdot)$ be a unique stationary solution to the following scalar equation

$$dz + zdt = dW_t \tag{2.7}$$

Then, by putting $v(t) = e^{-z(\theta_t\omega)}u(t)$ and using Ito formula, we arrive at

$$\begin{aligned} de^{-z(\theta_t\omega)} &= \left(z(\theta_t\omega)e^{-z(\theta_t\omega)} + \frac{1}{2}e^{-z(\theta_t\omega)} \right)dt - e^{-z(\theta_t\omega)}dW_t \\ &= z(\theta_t\omega)e^{-z(\theta_t\omega)}dt - e^{-z(\theta_t\omega)}{}_\circ dW_t, \end{aligned} \tag{2.8}$$

where the second equality above follows from the conversion between the Ito and Stratonovich integrals. Furthermore, we have that

$$dv = d\left(e^{-z(\theta_t\omega)}u\right) = u_\circ de^{-z(\theta_t\omega)} + e^{-z(\theta_t\omega)}{}_\circ du. \tag{2.9}$$

Hence, Eq. (1.1) becomes

$$\frac{dv}{dt} + Av = z(\theta_t\omega)v + e^{-z(\theta_t\omega)} f(t, e^{z(\theta_t\omega)}v). \tag{2.10}$$

Next, by a *mild solution* to equation (2.10) on an interval $\mathbb{J}$ we mean a strongly measurable function $v(\cdot)$ defined on $\mathbb{J}$ with the values on X_θ that satisfies the integral equation

$$v(t) = e^{-(t-s)A + \int_s^t z(\theta_r\omega)dr} v(s) + \int_s^t e^{-(\tau-s)A + \int_s^\tau z(\theta_r\omega)dr - z(\theta_\tau\omega)} f\left(\tau, e^{z(\theta_\tau\omega)}v(\tau)\right)d\tau \tag{2.11}$$

for a.e. $t \ge s, t, s \in \mathbb{J}$ and $\omega \in \Omega$.

We then give the notion of inertial manifolds in the following definition.

Definition 2.5. A stochastic inertial manifold for mild solutions to Eq. (2.10) is a collection of Lipschitz surfaces $\{M(\omega)\}_{\omega\in\Omega}$ in X such that

(i) for each $\omega \in \Omega, M(\omega)$ can be represented as the graph of a Lipschitz mapping $m(\omega) : PX \to QX_\theta$, i.e., $M(\omega) = \{ x + m(\omega)x^{\cdot\cdot}\ x \in PX \};$;

(ii) there exists a constant $\gamma > 0$ such that to each $x_0 \in M(\omega)$ there corresponds one and only one solution $v(\cdot)$ to Eq. (2.11) on $(-\infty, 0]$ such that $v(0) = x_0$ and

$$\sup_{t\le 0} \left\| e^{\gamma t - \int_0^t z(\theta_r\omega)dr} A^\theta v(t) \right\| < \infty \tag{2.12}$$

(iii) $M(\omega)$ is positively invariant under Eq. (2.11), i.e., if a solution $v(t), t \ge 0$ of Eq. (2.11) satisfies $v(0) \in M(\omega)$, then we have $v(t) \in M(\theta_t\omega)$ for all $t > 0$;

(iv) $M(\omega)$ exponentially attracts all the solutions to Eq. (2.11), i.e., for any solution $v(\cdot)$ of Eq. (2.11) there exist a solution $v^*(\cdot)$ of Eq. (2.11) with $v^*(t)\in M(\omega)$ for all $t\geq 0$ and a constant $H(\omega)$ such that $\left\|A^{\theta}\left(v^*(t)-v(t)\right)\right\|\leq H(\omega)e^{-\gamma t}$ for $t>0$.

Lemma 2.6. *Let* $f:\mathbb{R}\times X_\theta\to X$ *be* φ*-Lipschitz for a positive function* φ *belonging to an admissible space* E *such that*

$$R(\varphi,\theta):=\sup_{t\in\mathbb{R}}\left(\int_{t-1}^{t}\frac{\varphi(s)^{\frac{1+\theta}{2\theta}}}{(t-s)^{\frac{1+\theta}{2}}}ds\right)<\infty. \tag{2.13}$$

Let $v(t), t\leq 0$, *be a solution to* (2.11) *such that* $v(t)\in X_\theta$ *for* $t\leq 0$ *and*

$$\sup_{t\leq 0}\left\|e^{\gamma t-\int_0^t z(\theta_r\omega)dr}A^{\theta}v(t)\right\|<\infty. \tag{2.14}$$

Then, $v(t)$ satisfies

$$v(t)=e^{-tA+\int_0^t z(\theta_r\omega)dr}\mu+\int_{-\infty}^{0}G(t,s)e^{\int_s^t z(\theta_r\omega)dr-z(\theta_s\omega)}f\left(s,e^{z(\theta_s\omega)}v(s)\right)ds \tag{2.15}$$

where $\mu\in PX$, and $G(t,s)$ is the Green function defined as in (2.4).

Proof. Put $y(t)=\int_{-\infty}^{0}G(t,s)e^{\int_s^t z(\theta_r\omega)dr-z(\theta_s\omega)}f\left(s,e^{z(\theta_s\omega)}v(s)\right)ds.$ (2.16)

We have $y(t)\in X_\theta$ for $t\leq 0$, and

$$\sup_{t\leq 0}\left\|e^{\gamma t-\int_0^t z(\theta_r\omega)dr}A^{\theta}y(t)\right\|\leq k\left(1+\sup_{t\leq 0}e^{\gamma t-\int_0^t z(\theta_r\omega)dr}\left\|A^{\theta}v(t)\right\|\right)<\infty \tag{2.17}$$

Where

$$k=\begin{cases}\dfrac{M\left(\theta^{\theta}N_1+\lambda_{N+1}^{\theta}N_1+\lambda_N^{\theta}N_2\right)}{1-e^{-\alpha}}\left\|\Lambda_1\varphi\right\|_\infty+M\theta^{\theta}R(\varphi,\theta)\left(\dfrac{1-\theta}{(1+\theta)\alpha}\right)^{\frac{1-\theta}{1+\theta}} & \text{for } 0<\theta<\dfrac{1}{2}\\[2ex] \dfrac{M(N_1+N_2)}{1-e^{-\alpha}}\left\|\Lambda_1\varphi\right\|_\infty & \text{for } \theta=0.\end{cases} \tag{2.18}$$

By computing directly, one can verify that $y(\cdot)$ satisfies the integral equation

$$y(0) = e^{tA-\int_0^t z(\theta_r\omega)dr} y(t) + \int_t^0 e^{sA-\int_0^s z(\theta_r\omega)dr - z(\theta_s\omega)} f\left(s, e^{z(\theta_s\omega)} v(s)\right) ds. \tag{2.19}$$

On the other hand,

$$v(0) = e^{tA-\int_0^t z(\theta_r\omega)dr} v(t) + \int_t^0 e^{sA-\int_0^s z(\theta_r\omega)dr - z(\theta_s\omega)} f\left(s, e^{z(\theta_s\omega)} v(s)\right) ds.$$

Then

$$v(0) - y(0) = e^{tA-\int_0^t z(\theta_r\omega)dr} [v(t) - y(t)]. \tag{2.20}$$

We need to prove that $v(0) - y(0) \in PX$. To do this, applying the operator $A^\theta (I-P)$ to (2.20), we have

$$\left\| A^\theta (I-P)[v(0) - y(0)] \right\| = \left\| e^{tA-\int_0^t z(\theta_r\omega)dr} A^\theta (I-P)[v(t) - y(t)] \right\|$$

$$\leq M e^{(\lambda_{N+1} - \gamma)t} \|I - P\| \left\| e^{\gamma t - \int_0^t z(\theta_r\omega)dr} A^\theta [v(t) - y(t)] \right\|.$$

Since $\left\| e^{\gamma t - \int_0^t z(\theta_r\omega)dr} A^\theta [v(t) - y(t)] \right\| < \infty$, letting $t \to -\infty$ we obtain

$$\left\| A^\theta (I-P)[v(0) - y(0)] \right\| = 0,$$

Hence $A^\theta (I-P)[v(0) - y(0)] = 0.$

Since A^θ is injective, it follows that $(I-P)[v(0) - y(0)] = 0$.

Thus, $\mu = v(0) - y(0) \in PX$.

Since the restriction of e^{-tA} on PX, $t \geq 0$, is invertible with the inverse $e^{tA}P$, we have for $t \leq 0$ that

$$v(t) = e^{-tA+\int_0^t z(\theta_r\omega)dr} \mu + y(t)$$

$$= e^{-tA+\int_0^t z(\theta_r\omega)dr} \mu + \int_{-\infty}^0 G(t,s) e^{\int_s^t z(\theta_r\omega)dr - z(\theta_s\omega)} f\left(s, e^{z(\theta_s\omega)} v(s)\right) ds.$$

The proof is completed.

The following lemma describes the existence and uniqueness of solution belonging to weighted spaces.

Lemma 2.7. *Let* $f:\mathbb{R}\times X_\theta \to X$ *be* φ *-Lipschitz for* φ *satisfying the condition (2.13). Let the constant* k *be defined as in* (2.18). *Then, if* $k<1$, *there corresponds to each* $\xi \in PX$ *one and only one solution* $v(t)=v(t,\omega,\xi)$ *of Eq.(2.11) on* $(-\infty,0]$ *satisfying the condition* $v(0)=\xi$ *and* $\sup_{t\le 0} e^{\gamma t-\int_0^t z(\theta_r\omega)dr}\left\|A^\theta v(t)\right\|<\infty.$

Proof. We denote

$$L_\infty^{\gamma,-}=L_\infty^{\gamma}\left((-\infty,0],X_\theta\right)$$
$$=\{h:(-\infty,0]\to X_\theta \mid h \text{ is strongly measurable}$$
$$\text{and } \sup_{t\le 0} e^{\gamma t-\int_0^t z(\theta_r\omega)dr}\left\|A^\theta h(t)\right\|<\infty\}$$

endowed with the norm $\|h\|_{\gamma,-,\infty}=\sup_{t\le 0} e^{\gamma t-\int_0^t z(\theta_r\omega)dr}\left\|A^\theta h(t)\right\|.$

For each $\xi\in PX$, we define the transformation T as

$$(Tv)(t)=e^{-tA+\int_0^t z(\theta_r\omega)dr}\xi+\int_{-\infty}^0 G(t,s)e^{\int_s^t z(\theta_r\omega)dr-z(\theta_s\omega)} f\left(s,e^{z(\theta_s\omega)}v(s)\right)ds \text{ for } t\le 0.$$

By the following estimates,

$$\|Tv\|_{\gamma,-,\infty}\le M\lambda_N^\theta\|\xi\|+k\left(1+\sup_{t\le 0} e^{\gamma t-\int_0^t z(\theta_r\omega)dr}\left\|A^\theta v(t)\right\|\right),\forall v(\cdot)\in L_\infty^{\gamma,-},$$

and $\|Tu(\cdot)-Tv(\cdot)\|_{\gamma,-,\infty}\le k\| u(\cdot)-v(\cdot)\|_{\gamma,-,\infty},\ \forall\, v\in L_\infty^{\gamma,-}.$

We conclude that $T:L_\infty^{\gamma,-}\to L_\infty^{\gamma,-}$ is contraction since $k<1$. Thus, there exists a unique $v(\cdot)\in L_\infty^{\gamma,-}$ such that $Tv=v$.

By definition of T we have that $v(\cdot)$ is the unique solution in $L_\infty^{\gamma,-}$ of (2.11) for $t\le 0$.

Theorem 2.8. *Let* φ *belong to an admissible space* E *and satisfy condition* (2.13) *and let* f *be* φ *-Lipschitz. Suppose that*

$$k<1 \text{ and } \frac{M^3k\lambda_N^{2\theta}N_2}{(1-k)(1-e^{-\alpha})}\left\|\Lambda_1\varphi\right\|_\infty+k<1 \tag{2.21}$$

where k is defined as in (2.18).

Then, there exists a stochastic inertial manifold for mild solutions to Eq. (2.10).

Proof. For each $\omega \in \Omega$ we define the map $m(\omega): PX \to QX_\theta$ by

$$m(\omega)x = \int_{-\infty}^{0} e^{sA-\int_0^s z(\theta_r\omega)dr - z(\theta_s\omega)} (I-P) f(s, e^{z(\theta_s\omega)} v(s)) ds = (I-P)v(0) \tag{2.22}$$

where $v(.)$ is the unique solution of Eq. (2.11) in $L_\infty^{\gamma,-}$ satisfying $Pv(0) = x$.

(Note that Lemma 2.7 guarantees the existence and uniqueness of such a v). Furthermore, for each $\omega \in \Omega$ we put $M(\omega) = \{x + m(\omega)x : x \in PX\}$.

From the definition of $m(\omega)$ it follows that

$$M(\omega) = \{v_0 \in PX \text{ there exists a solution } v = v(t,\omega,v_0) \in L_\infty^{\gamma,-}((-\infty,0], X_\theta) \text{ of (2.11) with } v(0) = v_0\}.$$

Then, $M(\omega)$ satisfies all the properties of an inertial manifold from Definition 2.5.

Firstly, we show that $m(\omega)$ is Lipschitz continuous. Indeed, for x_1, x_2 belonging to PX one has

$$\begin{aligned} &\left\| A^\theta \left(m(\omega)x_1 - m(\omega)x_2 \right) \right\| \qquad (2.23)\\ &\le \int_{-\infty}^{0} \left\| A^\theta e^{sA-\int_0^s z(\theta_r\omega)dr} (I-P) \right\| \left(e^{-z(\theta_s\omega)} \left\| f(s, e^{z(\theta_s\omega)} v_1(s)) - f(s, e^{z(\theta_s\omega)} v_2(s)) \right\| \right) ds \\ &\le \int_{-\infty}^{0} \left\| A^\theta e^{sA-\int_0^s z(\theta_r\omega)dr} (I-P) \right\| \varphi(s) \left\| A^\theta \left(v_1(s) - v_2(s) \right) \right\| ds \\ &\le \int_{-\infty}^{0} e^{-\gamma s} \left\| A^\theta G(0,s) \right\| \varphi(s) e^{\gamma s - \int_0^s z(\theta_r\omega)dr} \left\| A^\theta \left(v_1(s) - v_2(s) \right) \right\| \le k \left\| v_1(\cdot) - v_2(\cdot) \right\|_{\gamma.\theta,\infty}. \end{aligned}$$

Next, we estimate $\left\| v_1(\cdot) - v_2(\cdot) \right\|_{\gamma.\theta,\infty}$. We have that

$$\begin{aligned} e^{\gamma t-\int_0^t z(\theta_r\omega)dr} \left\| A^\theta \left(v_1(t) - v_2(t) \right) \right\| &= \Bigg\| e^{\gamma t-\int_0^t z(\theta_r\omega)dr} A^\theta \Bigg(e^{-tA+\int_0^t z(\theta_r\omega)dr} (x_1 - x_2) \\ &+ \int_{-\infty}^{0} G(t,s) e^{\int_s^t z(\theta_r\omega)dr - z(\theta_s\omega)} \left[f(s, e^{z(\theta_s\omega)} v_1(s)) - f(s, e^{z(\theta_s\omega)} v_2(s)) \right] \Bigg) \Bigg\| \\ &\le M\lambda_N^\theta \left\| A^\theta (x_1 - x_2) \right\| + k \left\| v_1(\cdot) - v_2(\cdot) \right\|_{\gamma,-,\infty} \quad \text{for all } t \le 0. \end{aligned}$$

Hence, we obtain

$$\left\|v_1(\cdot)-v_2(\cdot)\right\|_{\gamma,-,\infty} \le M\lambda_N^{\theta}\left\|A^{\theta}(x_1-x_2)\right\| + k\left\|v_1(\cdot)-v_2(\cdot)\right\|_{\gamma,-,\infty}$$

and since $k<1$, we get $\left\|v_1(\cdot)-v_2(\cdot)\right\|_{\gamma,-,\infty} \le \frac{M\lambda_N^{\theta}}{1-k}\left\|A^{\theta}(x_1-x_2)\right\|.$

Substituting the above inequality into (2.23) we obtain

$$\left\|A^{\theta}\left(m(\omega)x_1-m(\omega)x_2\right)\right\| \le \frac{Mk\lambda_N^{\theta}}{1-k}\left\|A^{\theta}(x_1-x_2)\right\|.$$

Therefore, the property (i) in Definition 2.5 holds.

The property (ii) in Definition 2.5 follows from Lemma 2.7.

We then prove the property (iii). To do this, for each fixed $\omega\in\Omega, v_0\in M(\omega)$ and $t>0,$ let $v(\cdot)$ be a mild solution of (2.10) on $[0,t]$ with initial datum v_0 (in the fiber ω).

Put $\xi(s,\theta_t\omega, v(t,\omega,v_0)) = v(s+t,\omega,v_0)$ for all $s\le 0$.

Then, to prove $v(t)=v(t,\omega,v_0)\in M(\theta_t\omega)$ we will show that $\xi\in L_{\infty}^{\gamma,-}(\theta_t\omega)$.

This claim follows from the fact that

$$\begin{aligned}
&\sup_{s\le 0} e^{\gamma s-\int_0^s z(\theta_{r+t}\omega)dr}\left\|A^{\theta}\xi(s,\theta_t\omega,v(t,\omega,v_0))\right\| \\
&= \sup_{s\le 0} e^{\gamma s-\int_t^{s+t} z(\theta_r\omega)dr}\left\|A^{\theta}v(s+t,\omega,v_0)\right\| \\
&= \sup_{\tau\le t} e^{\gamma(\tau-t)-\int_t^{\tau} z(\theta_r\omega)dr}\left\|A^{\theta}v(\tau,\omega,v_0)\right\| \\
&= e^{-\gamma t+\int_0^t z(\theta_r\omega)dr}\sup_{\tau\le t} e^{\gamma\tau-\int_0^{\tau} z(\theta_r\omega)dr}\left\|A^{\theta}v(\tau,\omega,v_0)\right\| < \infty.
\end{aligned}$$

Therefore, the property (iii) in Definition 2.5 holds.

Lastly, we prove the property (iv). To this end, denote

$$L_{\infty}^{\gamma,+} = \{v:[0,+\infty)\to X_{\theta} \,|\, v \text{ is strongly measurable and } \sup_{t\ge 0} e^{\gamma t-\int_0^t z(\theta_r\omega)dr}\left\|A^{\theta}v(t)\right\| < +\infty\}.$$

Assume that $v^*(\cdot), v(\cdot)$ are two solutions of (2.10). Let $w=v^*-v$, then w is a solution to the equation

$$\frac{dw}{dt} + Aw = z(\theta_t\omega)w + e^{-z(\theta_t\omega)}F(t, e^{z(\theta_t\omega)}w) \tag{2.24}$$

where $F(t,e^{z(\theta_t\omega)}w) = f(t,e^{z(\theta_t\omega)}(u+w)) - f(t,e^{z(\theta_t\omega)}u).$

One can see that if $w \in L_\infty^{\gamma,+}$ solve (2.24) then w can be expressed by

$$w(t) = e^{-tA+\int_0^t z(\theta_r\omega)dr}Qw(0) + \int_0^{+\infty} G(t,s)e^{\int_s^t z(\theta_r\omega)dr - z(\theta_s\omega)}F(s,e^{z(\theta_s\omega)}w(s))ds. \tag{2.25}$$

Since $u^*(0) = u(0) + w(0) \in M(\omega)$, and u^* lies on M iff $Qu^*(0) = m(\omega)\left(Pu^*(0)\right)$ we have $Qw(0) = -Qu(0) + m(\omega)\left(Pu(0) + Pw(0)\right)$.

Substituting this equality into (2.25) we obtain

$$\begin{aligned} w(t) = e^{-tA+\int_0^t z(\theta_r\omega)dr}&\left[-Qu(0) + m(\omega)\left(Pu(0) + Pw(0)\right)\right] \\ &+ \int_0^{+\infty} G(t,s)e^{\int_s^t z(\theta_r\omega)dr - z(\theta_s\omega)}F(s,e^{z(\theta_s\omega)}w(s))ds. \end{aligned} \tag{2.26}$$

We now prove the existence of solution $w(\cdot) \in L_\infty^{\gamma,+}$ to the Eq. (2.26).

To do this, we show that the transformation T defined by

$$\begin{aligned} (Tx)(t) = e^{-tA+\int_0^t z(\theta_r\omega)dr}&\left[-Qu(0) + m(\omega)\left(Pu(0) + Px(0)\right)\right] \\ &+ \int_0^{+\infty} G(t,s)e^{\int_s^t z(\theta_r\omega)dr - z(\theta_s\omega)}F(s,e^{z(\theta_s\omega)}x(s))ds \quad \text{for } t \ge 0. \end{aligned} \tag{2.27}$$

acts from $L_\infty^{\gamma,+}$ into itself and is a contraction.

Indeed, for $x(\cdot) \in L_\infty^{\gamma,+}$ we have $\left\|F(t,e^{z(\theta_t\omega)}x(t))\right\| \le \varphi(t)e^{z(\theta_t\omega)}\left\|A^\theta x(t)\right\|$

and by putting $q(x) = \left[-Qu(0) + m(\omega)\left(Pu(0) + Px(0)\right)\right]$ we can estimate

$$\begin{aligned} e^{\gamma t - \int_0^t z(\theta_r\omega)dr}\left\|A^\theta (Tx)(t)\right\| &\le e^{\gamma t - \int_0^t z(\theta_r\omega)dr}\left\|A^\theta e^{-tA+\int_0^t z(\theta_r\omega)dr}q(x)\right\| \\ &+ e^{\gamma t - \int_0^t z(\theta_r\omega)dr}\left\|A^\theta \int_0^{+\infty} G(t,s)e^{\int_s^t z(\theta_r\omega)dr - z(\theta_s\omega)}F(s,e^{z(\theta_s\omega)}x(s))ds\right\| \\ &\le e^{\gamma t}\left\|A^\theta e^{-tA}q(x)\right\| + k\|x\|_{L_\infty^{\gamma,+}} \end{aligned} \tag{2.28}$$

$$e^{\gamma t}\left\|A^{\theta}e^{-tA}q(x)\right\| \le \left\|e^{\gamma t}A^{\theta}e^{-tA}\left(-Qu(0)+m(\omega)\left(Pu(0)\right)\right)\right\|$$
$$+\left\|e^{\gamma t}A^{\theta}e^{-tA}\left(m(\omega)\left((Pu(0)+Px(0)\right)-m(\omega)\left(Pu(0)\right)\right)\right\|$$

and $\le Me^{-(\lambda_{N+1}-\gamma)t}\left(\left\|A^{\theta}\left(-Qu(0)+m(\omega)\left(Pu(0)\right)\right)\right\|\right.$

$$\left.+\left\|A^{\theta}\left(m(\omega)\left(Pu(0)+Px(0)\right)-m(\omega)\left(Pu(0)\right)\right)\right\|\right)$$
$$\le M\eta + M\left\|A^{\theta}\left(m(\omega)\left(Pu(0)+Px(0)\right)-m(\omega)\left(Pu(0)\right)\right)\right\|$$

Where $\eta = \left\|A^{\theta}\left(-Qu(0)+m(\omega)\left(Pu(0)\right)\right)\right\|$. By Lipschitz property of $m(\omega)$ we have

$$\left\|A^{\theta}\left(m(\omega)\left(Pu(0)+Px(0)\right)-m(\omega)\left(Pu(0)\right)\right)\right\| \le \frac{Mk\lambda_N^{\theta}}{1-k}\left\|A^{\theta}Px(0)\right\|$$
$$\le \frac{Mk\lambda_N^{\theta}}{1-k}\left\|\int_0^{\infty} A^{\theta}e^{sA-\int_0^s z(\theta_r\omega)dr - z(\theta_s\omega)} PF(s,e^{z(\theta_s\omega)}x(s))ds\right\|$$
$$\le \frac{M^2k\lambda_N^{2\theta}N_2}{(1-k)(1-e^{-\alpha})}\left\|\Lambda_1\varphi\right\|_{\infty}\left\|x\right\|_{L_{\infty}^{\gamma,+}}.$$

where k is defined as in (2.18). Substituting these estimates into (2.27) we obtain $Tx \in L_{\infty}^{\gamma,+}$ and $\left\|Tx\right\|_{L_{\infty}^{\gamma,+}} \le M\eta + \left[\frac{M^3k\lambda_N^{2\theta}N_2}{(1-k)(1-e^{-\alpha})}\left\|\Lambda_1\varphi\right\|_{\infty} + k\right]\left\|x\right\|_{L_{\infty}^{\gamma,+}}$.

Therefore, T acts from $L_{\infty}^{\gamma,+}$ into itself. Now, using the fact that

$\left\|F(t,e^{z(\theta_t\omega)}w_1)-F(t,e^{z(\theta_t\omega)}w_2)\right\| \le \varphi(t)e^{z(\theta_t\omega)}\left\|A^{\theta}(w_1-w_2)\right\|$ and for $x,z \in L_{\infty}^{\gamma,+}$ we estimate

$$\left\|e^{\gamma t-\int_0^t z(\theta_r\omega)dr}A^{\theta}(Tx(t)-Tz(t))\right\|$$
$$\le \frac{Mk\lambda_N^{2}}{1-k}\left\|\int_0^{\infty} A^{\theta}e^{sA-\int_0^s z(\theta_r\omega)dr-z(\theta_s\omega)}P\left(F(s,e^{z(\theta_s\omega)}x)-F(s,e^{z(\theta_s\omega)}z)\right)\right\|ds$$
$$+\int_0^{\infty}\left\|e^{\gamma(t-s)}A^{\theta}G(t,s)\right\|\left\|e^{\gamma s-\int_0^s z(\theta_r)dr-z(\theta_s\omega)}\left(F(s,e^{z(\theta_s\omega)}x)-F(s,e^{z(\theta_s\omega)}z)\right)\right\|$$
$$\le\left[\frac{M^3k\lambda_N^{2\theta}N_2}{(1-k)(1-e^{-\alpha})}\left\|\Lambda_1\varphi\right\|_{\infty}+k\right]\left\|x(\cdot)-z(\cdot)\right\|_{L_{\infty}^{\gamma,+}}.$$

Hence, if $\dfrac{M^3 k\lambda_N^{2\theta} N_2}{(1-k)(1-e^{-\alpha})}\left\|\Lambda_1\varphi\right\|_\infty + k < 1$, then $T : L_\infty^{\gamma,+} \to L_\infty^{\gamma,+}$ is a contraction. Thus, there exists a unique $w(\cdot) \in L_\infty^{\gamma,+}$ such that $v^*(0) = u(0) + w(0) \in M(\omega)$ and

$$\left\|A^\theta\left(v^*(t) - v(t)\right)\right\| = \left\|A^\theta w(t)\right\| \le \frac{M\eta}{1-L} e^{-\gamma t + \int_0^t z(\theta_r\omega)dr}, \forall t \ge 0$$

$$\le \frac{M\eta}{1-L}\kappa(\omega)e^{-\gamma t}, \ \forall t \ge 0$$

$$\le H(\omega)e^{-\gamma t}, \quad \forall t \ge 0.$$

where $H(\omega) = \dfrac{M\eta}{1-L}\kappa(\omega)$, $L = \dfrac{M^3 k\lambda_N^{2\theta} N_2}{(1-k)(1-e^{-\alpha})}\left\|\Lambda_1\varphi\right\|_\infty + k.$

Therefore, $M(\omega)$ exponentially attracts every solution u of (2.10).

Remark 2.9. By the determination the constant k we have that, for $0 \le \theta < \frac{1}{2}$, the condition (2.21) is fulfilled if the following two conditions hold.

(i) the difference $\lambda_{N+1} - \lambda_N$ is sufficiently large, and/or

(ii) the norm $\left\|\Lambda_1\varphi\right\|_\infty = \sup_{t\in\mathbb{R}} \int_{t-1}^{t} \varphi(s)ds$ is sufficiently small.

3. Application to Chafee-Infante Equation

In this section we will apply our results to non-autonomous Chafee-Infante equation with multiplicative noise which has the form

$$\begin{cases} \dfrac{\partial u(t,x)}{\partial t} = \dfrac{\partial^2 u(t,x)}{\partial x^2} + ru(t,x) - b(t)u^3(t,x) + u(t,x)\dot{W}(t), \ t > 0, 0 < x < \pi \\ u(t,0) = u(t,\pi) = 0, t \in \mathbb{R}; u(0,x) = \phi(x) \ \ 0 < x < \pi. \end{cases} \tag{3.1}$$

We choose the Hilbert space $X = L_2[0,\pi]$, consider the linear operator $A : D(A) \to X$ defined by

$$D(A) = \left\{y \in X : y \text{ and } y' \text{ are absolutely continuous } y' \in X, y(0) = y(\pi) = 0\right\},$$

$$A(y) = -y'' - ry \ \ \forall y \in D(A).$$

Without loss off generality, we can assume $r < 1$ then A is a positive operator with discrete point spectrum being $1^2 - r, 2^2 - r, \cdots, n^2 - r, \cdots.$

Now, (3.1) can be expressed as the following abstract Cauchy problem.

$$\begin{cases} \dfrac{du(t,\cdot)}{dt} + Au(t,\cdot) &= f(t,u(t,\cdot)) + u(t,\cdot)\dot{W}(t) \quad t>0 \\ u(0,\cdot) &= \phi(\cdot) \in X \end{cases} \tag{3.2}$$

where $f:\mathbb{R}\times X \to X$ is defined by $f(t,\phi)(x) := -b(t)\phi^3(x) \ \forall x \in (0,\pi)$.

By "cut-off" technique we next modify the equation (3.2) into modified one in which we can apply our results.

Concretely, for any fixed $\rho > 0$ we denote by $B_\rho := \{v \in X : \|v\| \le 1\}$ the ball with radius ρ in X. One can see that $\|f(t,u) - f(t,v)\| \le 3\rho^2 b(t)\|u-v\| \ \ \forall u,v \in B_\rho$.

Now, let $\chi(s)$ be an infinitely differentiable function on $[0,+\infty)$ such that

$\chi(s) = 1, \ \ 0 \le s \le 1; \ \ \chi(s) = 0, \ \ s \ge 2; \ 0 \le \chi(s) \le 1, \ \ |\chi'(s)| \le 2, \ \ s \in [0,+\infty)$

and consider a mapping $G:\mathbb{R}\times X \to X$ such that

$$G(t,u)(x) = \chi\left(\frac{2}{\rho}\|u\|\right) f(t,u)(x) \ \ \forall u \in D(A).$$

Then G is φ - Lipschitz with $\varphi(t) = 3\rho(5\rho+4)b(t)$. Indeed, for any $u,v \in D(A)$

If $u,v \in B_\rho$, we have

$$\begin{aligned} \|G(t,u) - G(t,v)\| &= \left\| \chi\left(\frac{2}{\rho}\|u\|\right) f(t,u) - \chi\left(\frac{2}{\rho}\|v\|\right) f(t,v) \right\| \\ &\le \left\| \chi\left(\frac{2}{\rho}\|u\|\right) f(t,u) - \chi\left(\frac{2}{\rho}\|v\|\right) f(t,u) \right\| + \left\| \chi\left(\frac{2}{\rho}\|v\|\right) f(t,u) - \chi\left(\frac{2}{\rho}\|v\|\right) f(t,v) \right\| \\ &\le 2\frac{2}{\rho}\big|\|u\| - \|v\|\big|.\|f(t,u)\| + \|f(t,u) - f(t,v)\| \\ &\le 3\rho^2 b(t)\left(\frac{4}{\rho}(1+\rho)+1\right)\|u-v\| \\ &\le \varphi(t)\|u-v\|; \end{aligned}$$

if $u \in B_\rho$ and $v \notin B_\rho$, we have

$$\begin{aligned} \|G(t,u) - G(t,v)\| &= \left\| \chi\left(\frac{2}{\rho}\|u\|\right) f(t,u) - \chi\left(\frac{2}{\rho}\|v\|\right) f(t,v) \right\| \\ &\le \left\| \chi\left(\frac{2}{\rho}\|u\|\right) f(t,u) - \chi\left(\frac{2}{\rho}\|v\|\right) f(t,u) \right\| \\ &\le 2\frac{2}{\rho}\big|\|u\| - \|v\|\big|.\|f(t,u)\| \\ &\le 3\rho^2 b(t)\frac{4}{\rho}(1+\rho)\|u-v\| \\ &\le \varphi(t)\|u-v\|; \end{aligned}$$

If $u, v \notin B_\rho$, we have $\|G(t,u) - G(t,v)\| = 0 \le \varphi(t)\|u - v\|$.

Thus, G is φ - Lipschitz with $\varphi(t) = 3\rho(5\rho + 4)b(t)$.

Now, we consider the following stochastic semilinear differential equation of abstract form

$$\begin{cases} \dfrac{du(t,\cdot)}{dt} + Au(t,\cdot) & = G(t,u(t,\cdot)) + u(t,\cdot)\dot{W}(t) \quad t > 0 \\ u(0,\cdot) & = \phi(\cdot) \in X. \end{cases} \tag{3.3}$$

The equation (3.3) is called the modified equation of (3.1) which defines the asymptotic behavior of original one.

Furthermore, $\gamma = \dfrac{\lambda_{N+1} - \lambda_N}{2} = \dfrac{2N+1}{2}$ is large enough when N is sufficiently large. Theorem 2.8 implies that if the norm $\|\Lambda_1 \varphi\|_\infty = \sup_{t\in\mathbb{R}} \int_{t-1}^{t} \varphi(s)ds = \sup_{t\in\mathbb{R}} \int_{t-1}^{t} 3\rho(5\rho+4)b(s)ds$ is sufficiently small, then there exists an inertial manifold for mild solutions to Eq. (3.3). This is an inertial manifold for mild solutions of Eq. (3.1) which are staying in B_ρ as $t \to -\infty$.

4. Conclusion

By Theorem 2.8, we proved the existence of an inertial manifolds for a class of stochastic differential equations which relate to non-uniformly Lipschitzian nonlinearity. Furthermore, in Section 3 we presented an example to illustrate our results.

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INTEGRATING CRITICAL THINKING SKILLS INTO READING-WRITING PRACTICE IN AN EFL SETTING FOR FIRST YEAR STUDENTS

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Received: 30 March 2018/ Accepted: 11 June 2019/ Published: June 2019

Abstract: *This paper promotes the idea of integrating critical thinking skills in language learning in order to derive learning products in the process of reading and writing for English freshmen in Ha Tinh university. The important key principle here is applying higher order thinking skills in every stage of project done because these skills empower learners to create their own products. The study was conducted on the basis of instruments like seminar, questionnaire survey, observation and interviews. The results revealed that: (1) 6 critical thinking levels of Bloom (1956) revised by Anderson (2000) were effective in academic success; (2) these critical thoughts strengthen involvement, collaboration, motivation, new discovery, language retention, better communication with their peers, confidence, self direction, facilitate project designing, and connect academic work to real- life issues.*

Keywords: *Critical thinking skills, project, critical thinking levels*.

1. Introduction

Critical thinking skills are very important for academic success as well as for future professional success in the workplace in the 21st century. This reality has been recognized by P21 adherents and educators everywhere. In fact, hiring managers are looking for employees who can use skills as reasoning and creative thinking to conduct research, to handle making important decisions, to solve complex problems, to collaborate or to carry out a project. These skills will help students learn to think deeply about the subject matter, consult appropriate sources, weigh their options, take time to digest the information or to make intelligent judgements and decisions, and consider a variety of similar scenarios.

According to John Dewey (1910), integrated skills or thinking skills are defined as follows "…a curriculum aimed at building thinking skills would be a benefit not only to the individual learner, but to the community and to the entire democracy". However, the students with whom I have ever worked find it difficult integrating ideas or skills like discussions, group projects, class readings, class writings, etc. and thinking critically about what they

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discuss, what they read, or whatever they do. The inherent reason here is that my students are quite new about the term "critical thinking" surveyed from Seminar Evaluation Form. They are not trained to think so they have to struggle with critical thinking and are unable to apply what they have learnt. For example, they do not understand the texts with new words or words with multiple meanings. That's why they often have problems reading and writing clear English and, in particular, creating something new from the lesson is often a struggle. However, instead of thinking something new, they want to remain safe inside the box in traditional lessons. Therefore, how to get them out of the box and generate new ideas is a necessity. Do we need to wait for an apple to fall on your head or do we need some specific techniques to perceive something new, useful beforehand?

In order to answer this issue, ten critical projects were implemented at Ha Tinh university with freshmen English majors in a class of reading and writing skills. The idea behind the projects was to let students think cognitively out of the box through the application of principles such as cooperative work, discussion, planning, feedback, reflection, etc., allowing learners to take control of the learning process, and in particular, to take charge of their learning outcomes.

2. Literature review

2.1. What critical thinking?

The literature indicates that there is a contradiction regarding the definition of critical thinking. While some researchers consider critical thinking as a narrow concept, others deal with it as a broad concept. According to Beyer (1987), critical thinking is defined in a narrow sense as convergent thinking. Critical thinking, in his view, is convergent (p.35), different from creative thinking which is divergent. Convergent information or information that exists as that is the way they have always been ever before. By contrast, divergence refers to how to produce a greater number of complicated ideas from a single idea, a significant number of answers from a single question, and the like. The former seems not to be in accord with the usage of current critical thinking because it does not allow people to produce quality thinking that meets standards [3].

Cuseo (1996) points out that critical thinking or thinking deeply means that not only do people know the facts, but they also take the additional steps of going beyond the facts to do something with them. It is actually deeper thinking than memorization or recall of factual information. It involves reflecting the information received, moving far away from surface memorization or sifting away from viewing learning as the reception of information from teacher or textbook and toward deeper level of learning. Critical thinking includes some activities like making judgments about actions, beliefs, asking or answering questions, assessing the logic of statements or designing a creative project. In order to think creatively for an activity, there must always be a purpose for critical thoughts because as stated by McPeck (1990:3) "thinking as always thinking about something, or meta- cognition.".

Critical projects employed here needs both divergent thinking and convergent thinking because when the phase of divergent thinking like brainstorming, question- answer creation is complete, convergent thinking is used to organize information and new ideas for the proposal.

2.2. Why project work?

Compared to routine work, project work aims to provide language learners with opportunities to receive comprehensible input and produce comprehensible output" [5]. This means that learners are motivated to acquire the language not only as an academic subject, but as a tool for comprehension and performance in a meaningful foreign language context. Beckett & Slater (2005) and Stoller (1997) emphasizes that project work is said to be an effective way to promote the acquisition of language, content and skills simultaneously as it establishes a direct connection between language learning and its application" [12].

Beckett and Miller (2006) also add that the purpose of project work helps learners to recycle known language and skills in natural contexts. In fact, project work is filled with active learning in which learners can engage in authentic and interesting tasks for authentic purposes both of which are sadly absent from many language classrooms (Stoller, 2006:24) to reach a common goal by means of collaborative work. These collaborative tasks highlight the main characteristics of project- based learning which put emphasis on the learner and how they exercise their critical thinking skills. Learners are more likely to retain the knowledge through this approach more readily than through traditional textbook- centered learning.

2.3. Critical thinking and project work

P21 educators really recognize critical thinking as a foundation skill for the 21st century. By integrating project work into integrated- skill classes with certain topics such as reading and writing, teachers create vibrant learning environments that stimulate higher level thinking skills [16]. In order to do a successful project, learners need to make thoughtful decisions and exercise their reasoned judgments. For this to occur, they become critical thinkers. Due to comparing with "driving questions" which are insufficient enough to evoke careful thoughts, so do project tasks come in. The principle of project- based learning is for learners to learn something, they must do something. Therefore, project tasks designed must motivate learners' careful thoughts such as figuring out what is best to create something, making judgments between choices, weighing evidence, reconsidering initial ideas, etc. to help them develop their critical thinking competencies. Not only do critical thinking projects require learners to think carefully, but they scaffold and guide participants how cognitive tasks are carried out during the project. Regarding what mentioned above, project work can be understood as an efficient way to help learners become critical thinkers as it has the high output of critical thinking. As remarked by Beckett and Slater (2005), project based learning is a way to promote the simultaneous acquisition of language, content, and skills (p.108). Bloom's taxonomy (1956) which was modified by Anderson and Krathwohl (2000) including remembering, understanding, applying, analyzing, evaluating, and creating, will be applied in practicing cognitive skills in order to create the projects.

Bloom's Taxonomy Revised Version *(Anderson, L.W. et al., 2000)*

Level 1	**Remembering:** can the student recall or remember the information?	define, duplicate, list, memorize, recall, repeat, state.
Level 2	**Understanding:** can the student explain ideas or concepts?	classify, describe, discuss, explain, identify, locate, recognize, report, select, translate, paraphrase.
Level 3	**Applying**: can the student use the information in a new way?	choose, demonstrate, dramatize, employ, illustrate, interpret, operate, schedule, sketch, solve, use, write.
Level 4	**Analysing**: can the student distinguish between the different parts?	appraise, compare, contrast, criticize, differentiate, examine, experiment, question, test.
Level 5	**Evaluating**: can the student justify a stand or decision?	appraise, argue, defend, judge, support, evaluate.
Level 6	**Creating**: can the student create new product or point of view?	assemble, construct create, design, develop, formulate, write.

Research questions

What critical thinking skills are employed in order to empower learners' success of projects?

What is the role of critical thinking skills towards that academic success?

3. Methodology

3.1. Research design

In order to improve the students' critical thinking from projects, it would be helpful to conduct a classroom action research. It is necessary to do so because, according to Parsons and Brown (2002), action research is the appropriate research design to solve the students' problems and improve professional practices. Mattetal (2003) proved that action research is designed to help teachers know what is actually happening in classrooms and to use that knowledge to make decisions which are beneficial for the future. Kemmis and Mc. Taggart proposed that there are four key stages in the action research including planning, action, observation and reflection (1998:10).

This action research was conducted on 10 group projects, undertaken by 21 first year English major students at Ha Tinh university during the first semester of the integrated skills (reading and writing). During the development process, the participants were asked, in groups of three, to build new projects around the learning outcomes regarding 10 units of the thought- provoking Q series and new reading, vocabulary, grammar, and writing skills. The basis of the participants' projects in each unit was formed on considering new information of each aspect of learning as the heart of critical thinking approach. The projects provided students expectations of what they would study, what their teacher would teach. When they knew that they needed to learn content for a certain purpose, they would try to acknowledge new knowledge and skills, reinforce them, plan and complete their duty. Thus, the project work provided the focus that each lesson needed. The projects mentioned in this study were trend, color decoration (for festivals, children libraries, classrooms, clubs, etc.), good table manners, building professional sports teams, designing a family business website, describing a process, making art from trash, role-play, number importance in cultures, collaborating to complete the book “Destination B1”. Outstanding features of each project included:

The inclusion of six phases as described Anderson and comments in each project.

Participants came from two distinctive branches: English language and English pedagogy.

Self- formed groups were required at least 6 roles during each project from the following: innovator, explorer, harmonizer, Devil’s Advocate, Prioritizer, checker.

3.2. Instruments

It was an action research with both qualitative and quantitative methods have been employed to evaluate the result of the project experience in the light of critical thinking.

In the first part of the study, an evaluation form about the introductory seminar was done to survey the participants about the study field.

In order to measure the quantitative results, a Likert Scale was used to structure a questionnaire at the end of the semester when the project work finished which aimed to obtain information about the subjects’ opinions towards critical thinking approach including their motivation, their learning autonomy, their cooperative work, their learning strategies, their self- reliance. The questionnaire was composed of 24 statements scoring on a five- point scale for each (4= always, 3= often, 2= sometimes, 1=rarely, 0=never). The mentioned statements covered levels of critical thinking skills involved.

In the third part, student group interviews after each project presented consisted of 3 open- ended questions was used to collect respondents’ personal opinions about their critical project work. They are (1) Did you like building projects relating to steps done throughout each unit? Why?, (2) Did the activities of critical thinking skills help you to design your projects? What were they?, (3) Did you find something different between the lessons designed with projects and the traditional ones? What was it? This aimed to check how critical thinking had been developed into the project, how new knowledge from learning had been transferred to the project. Due to time limitation in the classroom, two participant groups were interviewed after every project.

Finally, in conduction of the research, the researcher also observed the participants’ use of skills associated with critical thinking through the levels of practices. The direct observations of all occasions of participants applying critical thinking in class were conducted through three consecutive units. Each unit was observed for 50 minutes accounting for one fourth of the total time of a unit (200 minutes). The observations took place when the participants were working in groups or when the activity ended in, or even as instructor’s home observation order to avoid stopping their progressive learning. The observers were both the instructor and participants as class or group secretary. To determine the reliability of recorded observations of using critical thinking skills, the core elements established for classroom observation consisted of observation steps, debriefing, action planning and follow- up.

4. Findings and discussion

To proceed the study, the instructor organized an introductory seminar of critical thinking skills aiming to survey participants' opinions as well as to form the idea of the study.

Table 1. *Critical Thinking Seminar Evaluation Results measured by Mean and Percentage*

Statements	No.	Mean	Percentage (%)				
			4	3	2	1	0
1. The content was as described in the textbook.	21	2.14	4.8	19	61.9	14.3	0
2. The seminar was applicable to my study.	21	2.38	0	57.1	33.3	0	9.5
3. Critical thinking was quite new.	21	3.81	95.2	0	0	0	4.8
4. The level was appropriate.	21	2.14	0	23.8	71.4	0	4.8
5. The handouts were helpful.	21	2.81	28.6	42.9	14.3	9.5	4.8
6. The seminar was effective.	21	2.81	23.8	47.6	19	4.8	4.8
7. The seminar was worth my time	21	2.76	19	57.1	9.5	9.5	4.8
8. The instructor had a good understanding of the topic.	21	2.95	23.8	66.7	0	0	9.5
9. I would be interested in attending more seminars on this same subjec.	21	2.81	23.8	61.9	0	0	14.3

(Interpreting numbers in italic*: 4 means* ***"strongly agree****", 3 means* ***"agree"****, 2 means* ***"disagree"****, 1 means* ***"strongly disagree****", 0 means "***no opinion***"; **Interpreting key to averages**: 2.5 or higher = agree or strongly agree; 2.4- 0.8 = disagree or strongly disagree; 0.7 or lower = no opinion).*

Overall, the total of respondents (21) to the initial survey positively commented on the introductory seminar of critical thinking and its use for designing projects although critical thinking- its concept is quite new with them.

Most of the participants agreed that the seminar was effective in terms of content, accounting for over 47% (agree) and nearly 24 % (strongly agree). More than half of participants stated that it was worth attending the seminar (57.1%) because it was perfect about materials and presentation with 42.9 and 66.7% respectively so more than 60% of them really want to attend more seminars on the same subject.

Even though the content was highly evaluated during the seminar, most participants considered it as a new concept (95.2%) so they found some activities not concerned with what the text says (over 60%). However, a significant number agreed that the seminar was applicable to their study which outweighed the number of participants who disagreed (over 30%). The success of the seminar proved that participants would accept the idea of teaching discussed.

Regarding the first research question, "What critical thinking skills are employed in order to empower learners' success of projects?" a taxonomy revised Anderson from (...) was applied for each unit including (1) remembering, (2) understanding, (3) applying, (4) analysing, (5) evaluating, (6) creating. For the second research question, "What is the role of critical thinking skills towards that academic success?", three categories were identified. In order to make these clear, the two research questions will be analyzed separately.

Table 2. *Percentage of Respondents Responding to Six Levels of Critical Thinking*

Skills surveyed	No.	Mean	Percentage				
			4	3	2	1	0
Level 1: Remembering							
1. It is important to recall information.	21	3.62	71.4	19	9.5	0	0
2. I know how to get multiple ideas.	21	3.10	23.8	61.9	14.3	0	0
3. I listen to other's ideas even I disagree with them.	21	2.48	9.5	28.6	61.9	0	0
4. I am able to give ideas that support the Unit question.	21	2.0	0	14.3	71.4	14.3	0
Level 2: Understanding							
5. I think of related words and ideas before reading.	21	3.19	23.8	71.4	4.8	0	0
6. I identify if my ideas are in the reading.	21	1.62	0	14.3	47.6	23.8	14.3
7. I know how to locate answers in the text.	21	2.76	14.3	57.1	19	9.5	0
8. I am able to explain my answers.	21	2.90	19	61.9	9.5	9.5	0

9. I identify and discuss reading skills.	21	2.62	4.8	57.1	33.3	4.8	0
10. I am able to discuss and report ideas after reading.	21	2.33	4.8	42.9	38.1	9.5	4.8
Level 3: Applying							
11. I am able to write using the ideas collected.	21	2.95	14.3	71.4	9.5	4.8	0
12. I know how to brainstorm more ideas.	21	2.76	14.3	47.6	38.1	0	0
13. I apply some vocabulary and structures for writing.	21	2.05	9.5	23.8	38.1	19	9.5
Level 4: Analysing							
14. I am able to analyze the reading text pattern and that of my writing.	21	2.71	9.5	61.9	19	9.5	0
15. I justify my thoughts, views through writing.	21	3.14	28.6	57.1	14.3	0	0
Level 5: Evaluating							
16. I know how to ask to check other's writing.	21	2.76	14.3	52.4	28.6	4.8	0
17. I review other's writing.	21	2.62	9.5	52.4	28.6	9.5	0
18. I revise my writing.	21	3.14	19	76.2	4.8	0	0
Level 6: Creating							
19. I compare ideas to select the best for projects.	21	2.90	14.3	66.7	14.3	4.8	0
20. I plan how to build a project.	21	3.48	47.6	52.4	0	0	0
21.I think about how to reach my goal.	21	2.52	9.5	33.3	57.1	0	0

22. I am ready to present any projects before class.	21	2.0	4.8	9.5	71.4	9.5	4.8
23. I am able to justify our opinions about projects.	21	2.19	4.8	28.6	52.4	9.5	4.8
24. Feedback from the instructor and class helps us to perfect our projects.	21	2.38	0	42.9	52.4	4.8	0

***(Interpreting numbers in italic:** 4 means "always", 3 means "often", 2 means "sometimes", 1 means "rarely", 0 means "never"; **Interpreting key to Mean score**: 2.5 or higher = always, often; 2.4- 1.6 = sometimes; 1.5 or lower = rarely or never).*

The sub-category related to the first research question identified firstly was recalling information which refers to the ability to evoke, remember or repeat information from long-term memory. The high mean scores show that most of participants were able to do activities concerning this foundation skill. By contrast, no one felt difficult to deal with this. Over 70% of subjects agreed that it is always important to recall information before reading; 61.9% often noticed getting diverse ideas from class, and the similar percentage was correct with what they sometimes did when they listened to the ideas of others that they disagreed with.

The second category refers to understanding skills. Although participants sometimes had difficulties connecting their own ideas with the new language in the reading (mean score of 1.62), they understood points, concepts and skills stated in the reading to locate, explain, or discuss their findings (mean scores of 2.76, 2.90, 2.33 respectively).

For the third set of skills refers to applying which involves transferring what was learned into a new one. Most of subjects applied their previous and new knowledge to establish writing skills of description, proposal, opinion, letter, narration and definition and create their own writing products with mean scores of 2.76, 2.05 and 2.95 respectively.

The sub-category identified fourthly was analysing referring to breaking down the reading and writing texts in order to understand every idea in each paragraph logically. A high mean score (3.14) for opinion justification and 2.71 for the ability of recognizing the patterns proved that most of participants are able to analyze types of texts.

The fifth identified category was evaluating relating to judging the credibility of their writing product after reading. Evaluated skills listed on the chart shows that a significant percentage of participants who often paid attention to checking, reviewing and revising the writing, accounting for more than half of the total percentage.

The last and highest order skill was creating which refers to building a certain project per unit. Most of subjects mastered necessary skills from planning to finishing their projects. Of all six mentioned skills belonging to creating, planning was considered as the most important one with mean score of 3.48. On the contrary, a variety of subjects were reluctant to present their projects (2.0). In general, critical thinking skills from level one to five provided opportunities for participants to level six by expressing their ideas in a new way through building imaginative projects. The mean scores among the six groups of critical thinking skills shown on the chart suggested that participants actually performed their cognitive thoughts well because there was seldom anyone refused to do the task. The data also showed that there was a logical development and a positive relationship among six level of critical thinking which helped participants to develop from foundation skills such as recalling, understanding to higher order thinking like creating products.

On the other hand, the following groups are related to the second research question: first, critical thoughts promote involvement, collaboration, motivation, new discovery, language retention, better communication with their peers, confidence, self direction; second, integration of critical thinking skills in such an authentic way can allow the instructor to facilitate project designing; third, practices of critical thinking allows to see how academic work can connect to real-life issues leading to success in life. The following excerpts from the interviews and the data from the questionnaire survey and classroom observation will illustrate the fact.

Table 3. *Students' responses in interviews*

No.	Question and answer
1	Did you like building projects related to steps done throughout each unit? Why? *S9: Yes, I feel more enjoyable and engaged because I can get out of the box which only guides me to follow and memorize. I have to think on feet and grammar also goes out of the window.* *S11: Sure, it is very interesting as I can create something new as project at the end of each unit.* *S14: Of course, yes, madam. I get a chance to challenge with my groupmates for an outcome.* *S21: Yes, I need following every step so that I and my groupmates can generate beautiful ideas for our project.*
2	Did the activities of critical thinking skills help you to design your projects? What were they? *S4: Of course, yes, I could raise any questions precisely to provoke divergent thinking from my partners.* *S6: Yes, teacher, asking questions to clarify a position was what I cared.* *S10: Yes, I saw multiple sides of an issue by debating with my partners in group discussions as well as in a class discussion.* *S15: Yes, I was able to create something new like projects that I and my friends tried.* *S17: Yes, I could communicate effectively with my partners in figuring out solutions for our projects.*
3	Did you find something different between the lessons designed with projects and the traditional ones? What was it? *S1: Yes, quite different. The lesson aimed to reach an outcome always requires us a lot of thinking while we often feel bored with the traditional one.* *S3: Yes, for example, being experienced types of family businesses through discussion and through the author's opinions in the text, what we could do is to create a website to advertise our future family restaurant. I never did so in other classes before.* *S5: Yes, now I can ask questions that ask my partners to give opinion or explain compared to basic question in class before.*

For the first group, cognitive thoughts enhanced participant involvement because they realized what came to class ready for a new project. They typically also empowered since they knew that what their product was like depends on knowledge they got from the critical levels. One interviewed participant replied: "*I need following every step so that I and my groupmates can generate beautiful ideas for our project*". In terms of achievement, most of respondents considered projects effective. It was worth their time to collaborate to achieve the goal set before the lesson. Factors as collaboration, motivation and better communication were discovered when most of subjects agreed that they *felt more engaged, more enjoyable, more interesting, more relevant* as they were offered chances to discover new ideas, to challenge each other for a better outcome. In addition, students develop confidence and self-direction as they move through both team-based and independent work.

For the second group, critical thinking skills developed from bottom to top formed learners knowledge from the most basic to the highest which facilitates learning and creating in learning process as shown in table 2. One subject commented that she developed her critical thinking because she can raise questions with high- order rank instead of low- level questions that she used to.

For the third group, practice actually helped learners to improve their ability to think critically. Lifelong learning becomes the key as the educated subjects may be inspiring to pursue a career while they are working to create something. Critical practices directs participants to go beyond what might be on the final exam because they are evaluated on the basis of the outcome. Most learners stated that their academic work could connect to real- life issues. For instance, when studying about the topic "Family business", participants created different types of family businesses such as family restaurant, family bookshop, etc. Critical practice in this way inspires students to obtain a deeper knowledge of the subjects they're studying and students are more likely to retain the knowledge gained through this approach far more readily than through traditional textbook-centered learning.

However, In order to gain success for academic work during the semester, the classroom observations conducted with the purpose of following action steps, debriefing, action planning and following- up. Focused observations of participants' attitudes to cognitive skills in class through activities were performed in pairs, groups, and class. The notes on what observed in cycle 1 from unit 2 to unit 3 show that although they felt a bit challenging to do designed activities, they tried to do so to finish their duty. The class atmosphere became boring and stressful. Soon after the observation, questions, comments, idea sharing were performed for reflection on the cycle. The instructor made notes of participants' sharing and carefully planned to design effective activities for next classes.

5. Conclusion

Practices of critical thinking skills can help to create a productive atmosphere in class and make sure students improve their abilities. Revised taxonomy by Anderson seems more appropriate for projects, based on the final stage of the process is the stage that students are able to create their projects successfully. A process including six levels involving different skills makes students more focused on realistic communication, more motivated and engaged in classroom activities. These cognitive skills have significant roles as they demand more of student's input. Students will have to collaborate in learning to bring the best outcome as projects. Projects integrating critical thinking skills enhance students' understanding of social, work and life skills, acceptance of other different views, confidence, tolerance and self- direction as they move through independent and group work. The study is a practical approach to the integration of skills as reading and writing as a subject in my university. It is noticed that during the process of working with language, the participants had to think, negotiate, communicate, make connections, look for appropriate ideas, build ideas to show content, complement written texts and design products. They actually work with language in action. The action took place according to the order of the six levels of critical thinking revised by Anderson (2000) leading to success when the participants produced their own products together with development of necessary thinking skills. Critical thinking and academic performance have significant relationship with each other. Due to the important role of critical thinking in enhancing professional competence, the study result implies that the students' critical thinking skills will be developed if this research is applied or continued.

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SUPPORTING STUDENTS IN TRANSLATING LITERARY WORKS THROUGH DESIGNED EXERCISES: AN IDEA IN TEACHING ENGLISH AND AMERICAN LITERATURE TO ENGLISH MAJORS

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Received: 7 September 2018/ Accepted: 11 June 2019/ Published: June 2019

Abstract: *The English and American Literature module causes many difficulties for students because of language barriers, especially metaphorical expressions. This study focuses on designing exercises that help students overcome those barriers to improve their ability in translating English metaphorical expression. The results show that the students appreciated these exercises and, at the same time, the test results of the experimental group were higher than the control group.*

Keywords: *English and American literature, metaphorical expressions, learning English.*

1. Introduction

Teaching literature to English major students is a hard work for any teacher of English although the benefits of it are undeniable. As it "offers a bountiful and extremely varied body of written materials which is "important" in the sense that it says something about fundamental human issues, and which is ending rather than ephemeral." (Collier & Slater, 1988: 5) Therefore, how to teach this subject well in the second language classroom puts forwards a challenge for teachers. Sell (1995), Short (1988), Thiong'o, Nwa (1986) also study about the teaching of literature in language classroom, showing the challenge that teachers and students have to face with. We bear in mind the idea that teaching English and American literature is to help students enrich their cultural knowledge, their language, and also help to summon the whole person (here the students) into the literary works. We also realize that our students have quite a lot of difficulties in understanding the content of the works thoroughly and mainly the difficulties come from the metaphorical expressions in the works. This is a prerequisite for us to design exercises to help students understand and translate them better, therefore, enforcing the ability in translating the whole work. Our application of exercises has proved to be useful in terms of the translation results as well as motivation creating for English majors at Hong Duc university. The results show positive results when we compare the translation assessment between the experimental and the control group.

2. A Brief description of the current teaching reality

It is clear that for a foreign language learner who learns a literary work, they have both a linguistic and cultural gap to bridge, so preparing them with something so that it is within

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their normal reading proficiency is ideal. For the author recited their works not dependent on any goals for any level of readers, the choice is left to the teacher who should apply his or her knowledge, skills tactfully in choosing the appropriate material for his students.

The activities that have been carried out in our literary classroom are various such as role play, improvisation, creative writing, discussion, visuals etc. When we discovered that our students have problems in understanding the works and the obstacles were the metaphorical expressions, we decided to design the exercises to help them understand it better so that they can have a better translation of the works. The focal points of the course are:

Help students understand the works.

Equip students with the knowledge of culture.

Develop students' personal dimensions, i.e. behavior, ideology etc.

Build up students' translation competence.

Help students give their ideas on a certain issue generated from the works.

3. Some cornerstones of the study

3.1. Metaphor

There are different approaches to metaphor, in this study, we look at them in the view of cognitive approach. Metaphor is often considered a linguistic convention which implicitly compares two things. According to Lakoff and Johnson (1980) , it is a part of language usage, therefore, it is a part of cognition. Further, metaphor is not merely cognitive it is also a linguistic, sociocultural, neural, and bodily phenomenon. Metaphors are created based on conceptual ground, which is termed conceptual metaphor, and therefore it is not only understood as a separated one but in association with a system of many other metaphors, together, all of them form what is called a "conceptual metaphor". The distinction between the concept of metaphor in traditional approach and CL is that, in CL perspective, metaphor is considered as understanding one conceptual domain in terms of another conceptual domain. For example "He was in the highest spirits after you left" may mean "very happy" when one status of emotion is put into the domain of space (low or high). Bear in mind those ideas, we draw out metaphors from the works in the course and build exercises from them to helps students can understand the language better.

3.2. Procedure of exercise building

First, we had to scrutinize into the work to define and pick out metaphorical expressions. After they were selected, they then were put into different types exercises with the purpose that they can help students understand those metaphors better in order to understand the works better. We designed exercises in three types.

Type one: Direct translation: This exercise gave students a list of metaphorical expressions and asked them to work in pairs to translate them before they translated the work. The purpose of this is to make students focus one and have a clear understanding of those expressions. For example, in the work: "The adventure of the cardboard box" (Conan Doyle), several metaphorical expressions were listed for students as follows:

Translate the following expressions.

1. First he dropped me.

2. Miss Cushing had come upon a subject on which she felt very deeply.

3. Holmes listened attentively to everything, throwing in a question from time to time.

4. He had caught her meddling, I suspect, and given her a bit of his mind.With this exercise, students see that the metaphorical expressions are highlighted and also they can have an opportunity to discuss with their friends and their teacher about the meaning of those.

Type two: Matching: This exercise asks students to match the English expressions with Vietnamese equivalents. Both of them were given to students beforehand. The Vietnamese expressions are mixed so that students have to work in groups to sort them out. For example, in the story “The model millionaire” (Oscar Wilde), the exercise can be put in this way:

Match the English expression on the left with the Vietnamese equivalent on the right.

1. He never said a brilliant or even an ill-natured thing in his life. 2. Hughie lived on two hundred a year that an old aunt allowed him. 3. What was a butterfly to do among bulls and bears? 4. That did not answer.	Anh được người cô già chu cấp cho số tiền hai trăm một năm để sinh nhai. Anh sống trên hai trăm một năm mà người cô già cho phép anh. Anh ta chưa bao giờ nói một điều gì sáng láng hoặc một điều xấu xa trong đời. Anh ta chưa bao giờ nói một điều gì thông minh hoặc một thứ tự nhiên ốm đau trong đời. Điều đó đã không trả lời. Một con bướm phải bay thế nào giữa những con bò và gấu? Người cô già cho phép anh sống trên hai trăm một năm. Nhằm nhò gì châu chấu đá xe? Nhởn nhơ vui chơi giữa bò và gấu? Nó đã không trả lời. Việc đó cũng chả đem lại ích lợi gì.

This exercise is similar to multiple choice exercise, however, it is different in the way that the Vietnamese expressions are mixed so that students would have to work together figure out what should be the best translation for the metaphor they are dealing with.

Type three: Multiple choice: In this exercise, students would be supplied with expressions and their meaning in English, students then had to find out which option was the best to fit with the metaphorical expression given. The following contains some sample questions from the work “A story of the bad little boy”. (Mark Twain)

Choose the best option A, B, C or D for the similar expression with the bold one.

1. She would be glad to lie down in the grave and **be at rest**.

A. relax B. sleep C. at home D. die peacefully

2. The world might be harsh and cold towards him when she **was gone**.

A. disappeared B.died C. went D.moved

3. She was not anxious **on Jim's account.**

A. because of Jim B. about Jim’s calculation C. for his nickname D. for his explanation

4. He didn't kneel down and rise up with a **light, happy heart**.

A. without trouble B. bright C. not heavy D. small

With this exercise, students also worked individually or in pairs to define the exact meaning of the phrase selected. The purpose is to help students understand clearly the English expressions which are challenging for them in dealing with the full text.

4. Methodology

The study was carried out within 14 weeks – a semester with a different treatment to the experimental group, K16B with 51 students, and a group without treatment (in this case the designed exercise), K16A with 5 students. The course contained 7 works with more then 30,000 words. First, we discovered our students' difficulties in learning the subject after teaching one week, a questionnaire was handed to get their ideas on the subject. After it was obvious that they were very much concerned with difficult metaphorical expressions, we then decided to designed exercises to help them understand them better. Then during the time of application, we had some informal interview with them and got ideas to modify out exercises. At the end of the term, we handed another questionnaire to check see what is our students' ideas of the exercises in terms of their helpfulness. We also compare the translation results between two groups.

5. Findings and discussions

In this section, we would like to present the results of the questionnaire related to students' ideas of the difficulties they face as, the translation results as well as their evaluation of the exercises designed.

5.1. Students ideas of the difficulties they face in the subject.

With four dimensions related to language challenges which were supposed by the teacher, namely vocabulary, sentence structure, tense combination, and metaphorical expressions. The results collected are shown in the following table.

Table 1. *Difficulties students face in terms of language in literary works*

Level of difficult / **Dimension**	**Very difficult**		**Difficult**		**Not very difficult**		**Easy**	
	Number	Ratio (%)	Number	Ratio (%)	Number	Ratio (%)	Number	Ratio (%)
Vocabulary	13	25.5	32	62.7	6	11.8	0	13
Sentential structure	10	19.6	32	62.7	9	17.6	0	10
Tense combination	1	2.0	18	35.3	30	58.8	2	1
Metaphorical expression	39	76.5	11	21.6	1	2.0	0	39

With vocabulary, among 51 students surveyed, a significant number, 13 students, equivalent to 25.5% selected the *very difficult* level, while a majority of the students chose *difficult* level as what they perceive in vocabulary, only 11.8% agreed upon the third choice, it is *not very difficult.*

At the same level of difficulty, vocabulary and sentence structure share similar proportion: 26 and 20 percent, respectively. Besides, with the *difficult* level, 22% of the students select metaphorical expressions at this level, while 64 and 62 percent of students thought that that vocabulary and sentential structure are of this level. Specially, no one chose the third and fourth levels, *not very difficult,* and *easy* for metaphors. All students realized that metaphorical expressions are difficult, if not very difficult. This was an outstanding result and it needs to have a treatment to deal with a phenomenon that no one thinks not very difficult. This fact is understandable as this language use (metaphor) is never easy, in many cases even for the native speakers, as it is the product of the author's creativity.

5.2. Results of the translation assessment

After 14 weeks applying the exercises designed for the experimental class, we then synthesize the translation works and compare the two groups: K16A control group and K16B experimental group, and the results are shown as follows.

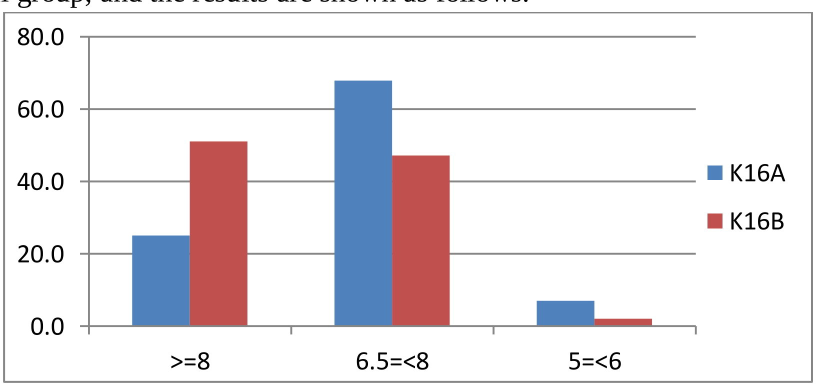

Figure 1. *Translation assessment results from the control and experimental group*

As can be seen from the chart, the marks from 6.5 to 8 in the control group are dominant (76%) while the number of the experimental group is 48%. Meanwhile, the experimental group outweighs the other in the rate of the mark 8 and higher. (50% compared to 28%). With the lowest mark group (from 5 to 6), the control group bears a higher number than the experimental one (6% compared to 2%). It is obvious that those who are helped with the metaphorical expressions beforehand have better accuracy in translating the work.

5.3. Students' evaluation of the designed exercises

After applying the exercises into the course, we then issued a questionnaire to understand how far we had helped students with the proposed difficult dimensions we mentioned at the beginning of the term. We also focused on four dimensions: vocabulary, sentence structure, tense combination, and metaphorical expressions. We only wanted to know if the exercises were helpful to the students. The results drawn out are shown in the following table.

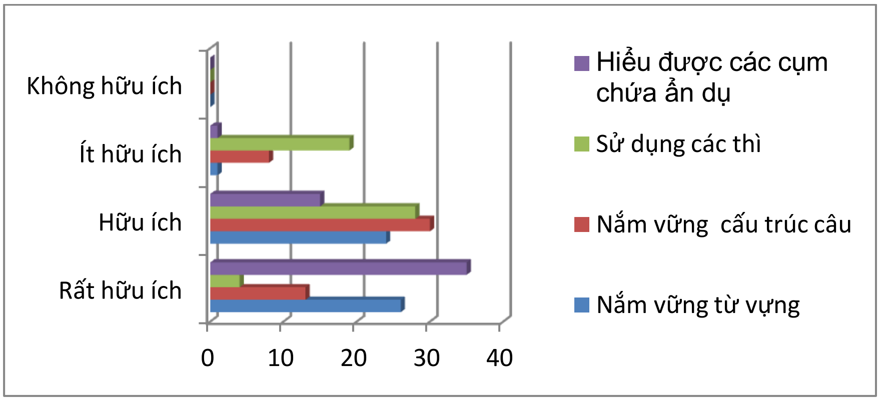

Figure 2. *Students' evaluation of the helpfulness of the exercises*

The table shows that among four level of helpfulness, 70 percent of students thought that the exercises were useful for them in understanding the metaphorical expressions, they also thought that the exercises were very helpful in learning vocabulary and structure (52 and 26 percent, respectively). Relatively equal numbers of students considered that those exercises were helpful for them in four dimensions, of which, the vocabulary, sentential structure and tense combination stand around 50 percent while 30 percent is the figure of metaphorical expressions. It can be seen that students think the exercises are very helpful and this is a good signal of what the teacher has carried out to help students understand the works in the course.

6. Conclusion

Teaching language to students is not an easy job, teaching language through literature is even more challenging. Being teachers, we know that, understanding clearly the language of the literary works would help students fulfill other goals we establish for the English majors in learning English and American literature, that is: Learning the language, the culture, perfect their personality etc. Beside several activities to teach the subjects which have been proved to be successful, designing exercises to help students understand clearly metaphorical expressions is a good way and has shown its significant effects in students' translation results as well as their positive feedback. Although it is not easy to design the exercises as it requires a lot of time and energy to design, it is worth doing so as we would come nearer to students' expectation.

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LONG-TIME BEHAVIOR OF SOLUTIONS TO A QUASILINEAR PARABOLIC EQUATION

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Received: 8 April 2019/ Accepted: 11 June 2019/ Published: June 2019

Abstract: *In this paper, we study the first initial boundary value problem for a class of quasilinear degenerate parabolic equations involving weighted p-Laplacian operators. The existence and uniqueness of a weak solution with respect to initial values is ensured by an application of the Faedo - Galerkin approximation and compact method. Moreover, the long-time behavior of solutions to that problem is considered via the concept of global attractors in various bi-spaces.*

Keywords: *Prabolic equation, bi-spaces.*

1. Introduction

Let $\Omega \subset \mathbb{R}^n$ $(n \geq 2)$ be a bounded open set with a sufficiently smooth boundary $\partial\Omega$. We are concerned with the following initial boundary value problem

$$\begin{cases} \dfrac{\partial u}{\partial t} - div\left(a(x)|\nabla u|^{p-2}\nabla u\right) + f(u) = g(x), x \in \Omega, t > 0, \\ u(x,t) = 0, x \in \partial\Omega, t > 0, \\ u(x,0) = u_0(x), x \in \Omega, \end{cases} \tag{1.1}$$

where the functions a, f, g satisfy

(H1) Let Σ be a closed subset of Ω such that $|\Sigma| = 0$. The function $a : \Omega \to \mathbb{R}$ satisfies the following conditions

i) $a(x) \in L^\infty(\Omega)$, ii) $a(x) = 0$ for $x \in \Sigma$,

iii) $a(x) > 0$ for $x \in \bar{\Omega} \setminus \Sigma$, iv) $\int_\Omega \frac{1}{[a(x)]^{\frac{n}{\alpha}}} dx < +\infty$ for some $\alpha \in \left(0, n(p-1)\right]$.

(H2) $f : \mathbb{R} \to \mathbb{R}$ is a continuously differentiable function satisfying

$$c_1|u|^q - c_0 \leq f(u)u \leq c_2|u|^q + c_0, q \geq 2, \tag{1.2}$$

$$f'(u) \geq -c_3, \textit{ for all } u \in \mathbb{R}, \tag{1.3}$$

where c_0, c_1, c_2, c_3 are positive constants.

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(H3) $g \in L^{s}(\Omega)$, where the positive number s is such that

$$\frac{s}{s-1} \in \left[1, p_{\alpha}^{*}\right),\ p_{\alpha}^{*} := \frac{pn}{n-p+\alpha},\ and\ 2 \le p < n+\alpha. \tag{1.4}$$

Recently, motivated by [7], where a semilinear degenerate elliptic problem was studied, the diffusion coefficient $a(x)$ is allowed to have at most a finite number of zeroes. Then, the attention is paid to a semilinear degenerate parabolic problem in [15] where its degeneracy is considered in the sense that the measurable, nonnegative diffusion coefficient $a(x)$ is allowed to be possibly vanished on a nonempty closed subset Σ with zero measure. For the physical motivation, it might be related to media model which possibly are somewhere *"perfect" insulators*or*"perfect" conductors*. So this allows the coefficient a vanish somewhere or to be unbounded (see [12]), or it might be related to the upper box-counting dimension of the set K when $a := dist(x; K)$ with K a subset of Ω (see [1]).

This paper is motivated by [4, 5, 13, 14, 21, 22] when we study the asymptotic behavior of the weak solutions of the problem by analysing the existence and structure of its global attractors. During the last decade, many mathematicians have been studing problems associated with the p-Laplace operator which appears in a variety of physical fields (see [2, 3, 4, 5, 6, 9, 10, 11, 13, 14, 21, 18, 20]). Recently, Yang, Sun and Zhong [2] proved the existence of an $(L^2(\Omega), W_0^{1,p}(\Omega) \cap L^q(\Omega))$- global attractor by using a new a *priori* estimate method to testify the asymptotic compactness.

The problem (1.1) contains some important classes of parabolic equations, such as the semilinear heat equation (when $a(x) = constant > 0, p = 2$), semilinear degenerate parabolic equations (when $p = 2$) which was investigated in [15], the p-Laplacian equation (when $a(x) = 1,\ p \ge 2$ see [21]), etc.

The paper is organized as follows. In Section 2, we prove the existence and uniqueness of a global weak solution to problem (1.1. In Section 3, we study the existence of global attractors in various bi-spaces for the semigroup.

The problem is distinguished in two cases such as subcritical if $\alpha \in \left(0, p + \frac{n(p-2)}{2}\right)$ and supcriticalif $\alpha \in \left[p + \frac{n(p-2)}{2}, n(p-1)\right]$.

This leads to the lack of a suitable compact embedding of $\mathcal{D}_0^{1,p}(\Omega,a)$ into $L^2(\Omega)$ in the supcritical case. Moreover, the solutions are at most in $\mathcal{D}_0^{1,p}(\Omega,a) \cap L^q(\Omega)$, so there is no compact embedding results for these cases which we need to prove the asymptotical compact for the semigroup. Therefore, the proof requires more involved techniques which makes it slightly complicated. In order to overcome these difficulties, we exploit the aproach in [5, 21, 22] which has been used recently for some kind of partial differential equations.

Notation: We use C to denote various constants whose values may change with each appearance. We write $\Omega(u \geq M) := \{x \in \Omega : u(x) \geq M\}$ and $\Omega(u \leq M) := \{x \in \Omega : u(x) \leq M\}$. By $\langle .,. \rangle$, we represent the both duality product and inner product. p', q' are conjugate of p, q, respectively.

2. Existence and uniqueness of weak solutions

First of all, let us introduce the energy space $\mathcal{D}_0^{1,p}(\Omega,a)$ defined as the closure of $C_0^\infty(\Omega)$ in the norm $\|u\|_{\mathcal{D}_0^{1,p}(\Omega,a)} := \left(\int_\Omega a(x)|\nabla u|^p \, dx\right)^{1/p}$. Let $\mathcal{D}^{-1,p'}(\Omega,a)$ be the dual space of $\mathcal{D}_0^{1,p}(\Omega,a)$. We denote

$$\Omega_T := \Omega\times(0,T),\ V := L^p\left(0,T;\mathcal{D}_0^{1,p}(\Omega,a)\right)\cap L^q(\Omega_T),\ V^* := L^{p'}\left(0,T;\mathcal{D}^{-1,p'}(\Omega,a)\right)+L^{q'}(\Omega_T).$$

Definition 2.1. A function u is called a weak solution of (1.1) on $(0,T)$ if and only if

$u \in V,\ \dfrac{\partial u}{\partial t} \in V^*,\ u|_{t=0} = u_0,\ a.e.\ in\ \Omega,$

and $\int_{\Omega_T}\left(\dfrac{d}{dt}u(t)\eta + a(x)|\nabla u|^{p-2}\nabla u\cdot\nabla\eta + f(u)\eta - g\eta\right)dxdt = 0$ for all test functions $\eta \in V$ and $u_0 \in L^2(\Omega)$.

It is known that (see [4]) that if $u \in V$ and $\dfrac{\partial u}{\partial t} \in V^*$, then $u \in C\left([0,T];L^2(\Omega)\right)$. This makes the initial condition in problem (1.1) meaningful. The following lemma is infered from Holder's inequality.

Lemma 2.1. *Under the assumption* ***(H1)****, the following embeddings holds*

$$(i)\ \ \mathcal{D}_0^{1,p}(\Omega,a) \subset W_0^{1,\beta}(\Omega)\ \text{continuously if}\ 1 \leq \beta \leq \frac{pn}{n+\alpha},$$

$$(ii)\ \ \mathcal{D}_0^{1,p}(\Omega,a) \subset\subset L^r(\Omega)\ \text{compactly if}\ 1 \leq r < p_\alpha^*.$$

By Young's inequality, embedding $\mathcal{D}_0^{1,p}(\Omega,a) \subset\subset L^{\frac{s}{s-1}}(\Omega)$ and Lemma 2.1, we have

Lemma 2.2. *Let* $\Omega' \subset \Omega$ *and* $u \in \mathcal{D}_0^{1,p}(\Omega',a)$, *we have*

$$\left|\int_{\Omega'} gudx\right| \leq \varepsilon\|u\|^p_{\mathcal{D}_0^{1,p}(\Omega',a)} + C(\varepsilon)\|g\|^{p'}_{L^s(\Omega')},\ \forall\varepsilon > 0.$$

Lemma 2.3. [9] *Let* $1 < p < +\infty$. *There exist positive constants* c_p, C_p *such that for every* $\xi, \eta \in \mathbb{R}^n$

$$c_p N_p(\xi,\eta) \le (|\xi|^{p-2}\xi - |\eta|^{p-2}\eta).(\xi-\eta) \le C_p N_p(\xi,\eta),$$

where $N_p(\xi,\eta) = \{|\xi| + |\eta|\}^{p-2}|\xi-\eta|^2$, *a dot denotes the Euclidean product in* $\mathbb{R}^n$.

Putting $L_{p,a}u := -div\left(a(x)|\nabla u|^{p-2}\nabla u\right)$.

As a consequence of Lemma 2.3 and using similar arguments as in [16, Chapter 2], we get

Lemma 2.4. *The operator* $L_{p,a}$ *maps* $\mathcal{D}_0^{1,p}(\Omega,a)$ *into its dual* $\mathcal{D}^{-1,p'}(\Omega,a)$. *Moreover,*

(i) $L_{p,a}$ *ishemicontinuous, i.e., for all* $u,v,w \in \mathcal{D}_0^{1,p}(\Omega,a)$, *the mapping* $\lambda \mapsto \langle L_{p,a}(u+\lambda v), w\rangle$ *is continuous from* $\mathbb{R} \to \mathbb{R}$.

(ii) $L_{p,a}$ *is strongly monotone when* $p \ge 2$, *i.e.,*

$$\langle L_{p,a}u - L_{p,a}v, u-v\rangle \ge \delta \|u-v\|^p_{\mathcal{D}_0^{1,p}(\Omega,a)}, \text{ for all } u,v \in \mathcal{D}_0^{1,p}(\Omega,a).$$

Lemma 2.5. *Let* $\{u_n\}$ *be a bounded sequence in* $L^p\left(0,T;\mathcal{D}_0^{1,p}(\Omega,a)\right)$.

Then $\{u_n\}$ *converges almost everywhere in* Ω_T *up to a subsequence.*

Proof. First, we prove that, for a.e. $t \in [0,T]$, there exists $C(t) > 0$ such that $\|u_n(t)\|_{\mathcal{D}_0^{1,p}(\Omega,a)} \le C(t)$.

Indeed, if there exists a set $\Lambda \subset [0,T]$ with positive measure such that $\|u_n(t)\|_{\mathcal{D}_0^{1,p}(\Omega,a)} \to \infty$ as $n \to \infty$, for all $t \in \Lambda$,

then $\int_0^T \|u_n(t)\|_{\mathcal{D}_0^{1,p}(\Omega,a)} dt \ge \int_\Lambda \|u_n(t)\|_{\mathcal{D}_0^{1,p}(\Omega,a)} dt \to \infty.$

This implies a contradiction. On the other hand, it follows from Lemma 2.1. that $\{u_n(t)\}$ is precompact in $L^r(\Omega)$ for some $r \in [1, p_\alpha^*)$. Therefore, $u_n(t) \to u(t)$ a.e. in Ω.

Theorem 2.1. *Given* $u_0 \in L^2(\Omega)$. *We assume that* (H1), (H2), *and* (H3) *hold. Then the problem (1.1) has a unique weak solution on the interval* $(0,T)$.

Moreover, the mapping $u_0 \mapsto u(t)$ *is continuous.*

Proof. **i) Existence.** Let $\{e_j\}_1^\infty$ be a basis of $\mathcal{D}_0^{1,p}(\Omega,a)\cap L^q(\Omega)$. We find the approximating solution $u_n(t)$ in the form $u_n(t)=\sum_{j=1}^{n} u_{nk}(t)e_k$.

We get u_n from solving the following problem

$$\int_\Omega \frac{du_n}{dt}e_k dx+\int_\Omega a(x)|\nabla u_n|^{p-2}\nabla u_n\cdot\nabla e_k dx+\int_\Omega f(u_n)e_k dx=\int_\Omega g(x)e_k dx, \tag{2.2}$$

$$\int_\Omega u_n(0)e_k dx=\int_\Omega u_0 e_k dx. \tag{2.3}$$

Since $f\in C^1(\mathbb{R})$ and using the Peano theorem, this problem possesses a local solution $u_{\{nk\}}(t)$. By multiplying by $u_{nk}(t)$ in (2.2) and summing k =1 to n, we obtain

$$\frac{1}{2}\frac{d}{dt}\|u_n\|_{L^2(\Omega)}^2+\int_\Omega a(x)|\nabla u_n|^p dx+\int_\Omega f(u_n)u_n dx=\int_\Omega g u_n dx. \tag{2.4}$$

We now establish some a *priori* estimates for u_n. Using (1.2) and Lemma 2.2 yield

$$\frac{1}{2}\frac{d}{dt}\|u_n\|_{L^2(\Omega)}^2+\|u_n\|_{\mathcal{D}_0^{1,p}(\Omega,a)}+c_1\|u_n\|_{L^q(\Omega)}^q\le c_0|\Omega|+\varepsilon\|u_n\|_{\mathcal{D}_0^{1,p}(\Omega,a)}+C(\varepsilon)\|g\|_{L^s(\Omega)}^{p'}, \tag{2.5}$$

for all $\varepsilon>0$.

It follows that

$$\frac{d}{dt}\|u_n\|_{L^2(\Omega)}^2+(2-2\varepsilon)\|u_n\|_{\mathcal{D}_0^{1,p}(\Omega,a)}+2c_1\|u_n\|_{L^q(\Omega)}^q\le 2c_0|\Omega|+2C(\varepsilon)\|g\|_{L^s(\Omega)}^{p'}. \tag{2.6}$$

After an integration in t, this leads to

$$\|u_n(t)\|_{L^2(\Omega)}^2+(2-2\varepsilon)\int_0^t\|u_n\|_{\mathcal{D}_0^{1,p}(\Omega,a)}dt+2c_1\|u_n\|_{L^q(\Omega_t)}^q\le\|u_n(0)\|_{L^2(\Omega)}^2+2tc_0|\Omega|+2tC(\varepsilon)\|g\|_{L^s(\Omega)}^{p'}. \tag{2.7}$$

We deduce from the last inequality that $\{u_n\}$ is bounded in $L^\infty(0,T;L^2(\Omega))$, $\{u_n\}$ is bounded in $L^p(0,T;\mathcal{D}_0^{1,p}(\Omega,a))$, $\{u_n\}$ is bounded in $L^q(\Omega_T)$.

We see that the local solution u_n can be extended to the interval $[0,T]$.

On the other hand, $L_{p,a}u_n$ defines an element of $\mathcal{D}^{-1,p'}(\Omega,a)$, determined by duality $\langle L_{p,a}u_n,w\rangle=\int_\Omega a(x)|\nabla u_n|^{p-2}\nabla u_n\cdot\nabla w dx$, for all $w\in\mathcal{D}_0^{1,p}(\Omega,a)$.

Taking (H1) into account and the boundedness of u_n in $L^p(0,T;\mathcal{D}_0^{1,p}(\Omega,a))$.

We deduce that $L_{p,a}u_n$ is bounded in $L^{p'}(0,T;\mathcal{D}^{-1,p'}(\Omega,a))$ since

$$\left|\int_0^T \langle L_{p,a}u_n, v\rangle\, dt\right| = \left|\int_0^T\int_\Omega a(x)|\nabla u_n|^{p-2}\nabla u_n\cdot\nabla v\, dxdt\right| \le \int_0^T\int_\Omega \left(a(x)^{\frac{p-1}{p}}|\nabla u_n|^{p-1}\right)\left(a(x)^{\frac{1}{p}}|\nabla v|\right)dxdt$$

$$\le \|u_n\|^{p/p'}_{L^p(0,T;\mathcal{D}_0^{1,p}(\Omega,a))}\|v\|_{L^p(0,T;\mathcal{D}_0^{1,p}(\Omega,a))},$$

for any $v\in L^p(0,T;\mathcal{D}_0^{1,p}(\Omega,a))$. In addition, from (1.2), we deduce

$$|f(u)|\le C(|u|^{q-1}+1). \tag{2.8}$$

Combining with the boundedness of $\{u_n\}$ in $L^q(\Omega_T)$ it implies that $\{f(u_n)\}$ is bounded in $L^{q'}(\Omega_T)$. We can rewrite the first equation of (1.1) in V^* as

$$u'_n = g(x) - L_{p,a}u_n - f(u_n). \tag{2.9}$$

Therefore, $\{u'_n\}$ is bounded in V^*. Due to Alaoglu weak-star compactness theorem (see [18])

$$u'_n \rightharpoonup u' \text{ in } V^*,$$

$$L_{p,a}u_n \rightharpoonup \xi_1 \text{ in } L^{p'}(0,T;\mathcal{D}^{-1,p'}(\Omega,a)), \tag{2.10}$$

$$f(u_n)\rightharpoonup \xi_2 \text{ in } L^{q'}(\Omega_T), \tag{2.11}$$

for all $T>0$.

Thanks to Lemma 2.5 and Lemma 1.3, p.12 in [16] and due to $f(u_n)$ is continuous and bounded.

We have
$$f(u_n)\rightharpoonup \xi_2 = f(u) \text{ in } L^{q'}(\Omega_T). \tag{2.12}$$

We now show that $\xi_1 = L_{p,a}u$. It follows from Lemma 2.4 that

$$X_n := \int_0^T \langle L_{p,a}u_n - L_{p,a}v, u_n - v\rangle\, dt \ge 0, \text{ for every } v\in L^p(0,T;\mathcal{D}_0^{1,p}(\Omega,a)).$$

Moreover, we have

$$\int_0^T \langle L_{p,a}u_n, u_n\rangle\, dt = \int_0^T\int_\Omega a(x)|\nabla u_n|^p\, dxdt = \int_0^T\int_\Omega (gu_n - f(u_n)u_n - u'_n u_n)\,dxdt \tag{2.13}$$

$$= \int_0^T\int_\Omega (gu_n - f(u_n)u_n)\,dxdt + \frac{1}{2}\|u_n(0)\|^2_{L^2(\Omega)} - \frac{1}{2}\|u_n(T)\|^2_{L^2(\Omega)}.$$

Therefore,

$$X_n = \int_0^T\int_\Omega (gu_n - f(u_n)u_n)\,dxdt + \frac{1}{2}\|u_n(0)\|^2_{L^2(\Omega)} - \frac{1}{2}\|u_n(T)\|^2_{L^2(\Omega)} = -\int_0^T\langle L_{p,a}u_n, v\rangle dt - \int_0^T\langle L_{p,a}v, u_n - v\rangle dt.$$

Since $u_n(0)\to u_0$ in $L^2(\Omega)$, and by the lower semi-continuity $\|\cdot\|_{L^2(\Omega)}$ we get

$$\|u(T)\|_{L^2(\Omega)} \le \liminf_{n\to+\infty}\|u_n(T)\|_{L^2(\Omega)}.$$

Meanwhile, by the Lebesgue dominated theorem, we have

$$\int_0^T \int_\Omega (gu - f(u)u)dxdt = \lim_{n\to+\infty} \int_0^T \int_\Omega (gu_n - f(u_n)u_n)dxdt. \tag{2.14}$$

Putting this together with (2.13) and (2.14), we obtain

$$\lim_{n\to+\infty} \sup X_n \le \int_0^T \int_\Omega (gu - f(u)u)dxdt + \frac{1}{2}\| u(0)\|^2_{L^2(\Omega)} - \frac{1}{2}\| u(T)\|^2_{L^2(\Omega)} - \int_0^T \langle \xi_1, v\rangle dt - \int_0^T \langle L_{p,a}v, u-v\rangle dt. \tag{2.15}$$

In view of (2.9), we deduce

$$\int_0^T \int_\Omega (gu - f(u)u)dxdt + \frac{1}{2}\| u(0)\|^2_{L^2(\Omega)} - \frac{1}{2}\| u(T)\|^2_{L^2(\Omega)} = \int_0^T \langle \xi_1, u\rangle dt.$$

Taking (2.15) into account, we get the estimate

$$\int_0^T \langle \xi_1 - L_{p,a}v, u-v\rangle\, dt \ge 0, \forall v \in L^p(0,T;\mathcal{D}_0^{1,p}(\Omega,a)).$$

We choose $v = u - \delta w,\ \delta > 0$, so

$$\int_0^T \langle \xi_1 - L_{p,a}(u-\delta w), w\rangle dt \ge 0, \quad \forall w \in L^p(0,T;\mathcal{D}_0^{1,p}(\Omega,a)).$$

Letting $\delta \to 0$, we get $\int_0^T \langle \xi_1 - L_{p,a}u, w\rangle dt = 0, \forall w \in L^p(0,T;\mathcal{D}_0^{1,p}(\Omega,a))$,

Thus $\xi_1 = L_{p,a}u$. We now prove $u(0) = u_0$.

By taking the test functions $\varphi \in C^1([0,T];\ \mathcal{D}_0^{1,p}(\Omega,a) \cap L^q(\Omega))$ such that $\varphi(T) = 0$,

we have $-\int_0^T \langle u_n, \varphi'\rangle dt + \int_0^T \langle L_{p,a}u_n, \varphi\rangle dt + \int_0^T \langle f(u_n) - g, \varphi\rangle dt = \langle u_n(0), \varphi(0)\rangle.$

Let $n \to \infty$, we obtain

$$-\int_0^T \langle u, \varphi'\rangle dt + \int_0^T \langle L_{p,a}u, \varphi\rangle dt + \int_0^T \langle f(u) - g, \varphi\rangle dt = \langle u_0, \varphi(0)\rangle, \tag{2.16}$$

since $u_n(0) \to u_0$. On the other hand, from the " limiting equation", we have

$$-\int_0^T \langle u, \varphi'\rangle dt + \int_0^T \langle L_{p,a}u, \varphi\rangle dt + \int_0^T \langle f(u) - g, \varphi\rangle dt = \langle u(0), \varphi(0)\rangle. \tag{2.17}$$

Comparing (2.16) with (2.17), we get $u(0) = u_0$.

ii) Uniqueness and continuous dependence on the initial data. Let us denote by u and v two weak solutions of (1.1) with initial data $u_0, v_0 \in L^2(\Omega)$, respectively. Then $w := u\text{-}v$ satisfies

$$\begin{cases} w_t + L_{p,a}\, u - L_{p,a}\, v + f(u) - f(v) = 0, \\ w|_{\partial\Omega} = 0, \\ w(0) = u_0 - v_0. \end{cases}$$

Hence $\frac{1}{2}\frac{d}{dt}\| w\|^2_{L^2(\Omega)} + \langle L_{p,a}u - L_{p,a}v, u-v\rangle + \int_\Omega \big(f(u) - f(v)\big)(u-v)dx = 0.$ By using (1.3) and Lemma 2.4 $\frac{d}{dt}\| w\|^2_{L^2(\Omega)} \le 2c_3\| w\|^2_{L^2(\Omega)}$.

An application of the Gronwall inequality leads to $\|w\|^2_{L^2(\Omega)} \le \|w(0)\|^2_{L^2(\Omega)} e^{2c_3 t}$. This implies the uniqueness (if $u_0 = v_0$) and the continuous dependence of the solution.

3. Global attractors

Theorem 2.1 allows us to construct a continuous (nonlinear) semigroup $S(t): L^2(\Omega) \to L^2(\Omega)$ associated to problem (1.1) as follows $S(t)u_0 := u(t)$, where $u(t)$ is the unique weak solution of (1.1) with the initial data u_0.

The exponent p^*_α plays a crucial role in the classical Sobolev embedding, i.e., $p^*_\alpha > 2$ when $\alpha \in (0, p + \frac{n(p-2)}{2})$, $p^*_\alpha \in [\frac{n}{n-1}, 2]$ *when* $\alpha \in [p + \frac{n(p-2)}{2}, n(p-1)]$. The main objective of this section is to show the existence of global attractors of the semigroup $S(t)$ generated by problem (1.1) in various bi-spaces. We will combine the so-called uniformly compact method and the method introduced in [5], [21], [22] to solve this problem. The following Proposition is the existence of bounded absorbing set.

Proposition 3.1. *The semigroup* $\{S(t)\}_{t\ge 0}$ *has an* $(L^2(\Omega), \mathcal{D}^{1,p}_0(\Omega,a) \cap L^q(\Omega))$-*bounded absorbing set* B_0, *i.e., there is a positive constant* ρ, *such that for any bounded subset* B *in* $L^2(\Omega)$, *there is a positive constant* T *which depends only on* L^2 *– norm of* B *such that* $\int_\Omega a(x)|\nabla u|^p dx + \int_\Omega |u|^q dx \le \rho$, *for all* $t \ge T$ *and where* u *is the unique weak solution of (1.1) with the initial datum* u_0.

Proof. Multiplying the first equation in (1.1) by u and integrating by parts, we have

$$\frac{1}{2}\frac{d}{dt}\|u\|^2_{L^2(\Omega)} + \|u\|^p_{\mathcal{D}^{1,p}_0(\Omega,a)} + \int_\Omega f(u)u dx = \int_\Omega g u dx. \tag{3.1}$$

Combining with (1.2) and using Lemma 2.2 yields

$$\frac{d}{dt}\|u\|^2_{L^2(\Omega)} + (2-2\varepsilon)\|u\|^p_{\mathcal{D}^{1,p}_0(\Omega,a)} + 2c_1\|u\|^q_{L^q(\Omega)} \le 2c_0|\Omega| + 2C(\varepsilon)\|g\|^{p'}_{L^s(\Omega)},$$

for ε small enough. Due to $q \ge 2$, we deduce from the last inequality that

$$\frac{d}{dt}\|u\|^2_{L^2(\Omega)} + C\|u\|^2_{L^2(\Omega)} \le C(\|g\|_{L^s(\Omega)}, c_0, |\Omega|). \tag{3.2}$$

Applying the Gronwall inequality, we obtain

$$\|u(t)\|^2_{L^2(\Omega)} \le \|u(0)\|^2_{L^2(\Omega)} e^{-Ct} + C(\|g\|_{L^s(\Omega)}, c_0, |\Omega|)(1-e^{-Ct}). \tag{3.3}$$

We see that, from (3.3), $\{S(t)\}_{t\geq 0}$ has an $(L^2(\Omega),L^2(\Omega))$-bounded absorbing set, i.e., for any bounded set B in $L^2(\Omega)$ there exists $T_1=T_1(B)$ such that $\|S(t)u_0\|^2_{L^2(\Omega)}\leq \rho_0$, (3.4)

for all $t\geq T_1, u_0\in B$, where the constant ρ_0 is independent of u_0. Going back to (3.1) and integrating over $[t,t+1]$ with $t\geq T_1$, we derive

$$\int_t^{t+1}\left(\|u\|^p_{\mathcal{D}_0^{1,p}(\Omega,a)}+\int_\Omega f(u)udx-\int_\Omega gudx\right)ds\leq \frac{\rho_0}{2}. \tag{3.5}$$

for all $t\geq T_1$. Putting $F(u)=\int_0^u f(s)ds$. Due to (1.2) and (1.3), it fulfills the bounds for some positive constants c_4, c_5 such that $c_4|u|^q-c_5\leq F(u)\leq f(u)u+\frac{c_3}{2}|u|^2$. (3.6)

$$\text{Therefore } c_4\|u\|^q_{L^q(\Omega)}-c_5|\Omega|\leq \int_\Omega F(u)dx\leq \int_\Omega f(u)udx+\frac{c_3\rho_0}{2}, \tag{3.7}$$

for all $t\geq T_1$. We deduce from (3.5) and (3.7) that

$$\int_t^{t+1}[\|u\|^p_{\mathcal{D}_0^{1,p}(\Omega,a)}+\int_\Omega F(u)dx-\int_\Omega gudx]ds\leq \frac{\rho_0(c_3+1)}{2}. \tag{3.8}$$

On the other hand, multiplying (1.1) by u_t, we obtain

$$\int_\Omega a(x)|\nabla u|^{p-2}\nabla u.\nabla u_t dx+\int_\Omega f(u)u_t dx-\int_\Omega g u_t dx\leq -\|u_t\|^2_{L^2(\Omega)}\leq 0. \tag{3.9}$$

$$\text{Therefore } \frac{d}{dt}\left(\frac{1}{p}\|u\|^p_{\mathcal{D}_0^{1,p}(\Omega,a)}+\int_\Omega F(u)dx-\int_\Omega gudx\right)\leq -\|u_t\|^2_{L^2(\Omega)}\leq 0.$$

Combining with (3.8) and (3.9), by virtue of the uniform Gronwall inequality, we get

$$\frac{1}{p}\|u\|^p_{\mathcal{D}_0^{1,p}(\Omega,a)}+\int_\Omega F(u)dx-\int_\Omega gudx\leq \frac{\rho_0(c_3+1)}{2}, \tag{3.10}$$

for all $t\geq T_2=T_1+1$. Thanks to (3.7) and Lemma 2.2, we infer from (3.10) that

$$\int_\Omega a(x)|\nabla u|^p\,dx+\int_\Omega |u|^q dx\leq C(\|g\|_{L^s(\Omega)},|\Omega|,\rho_0,p,c_3,c_4,c_5). \tag{3.11}$$

Thus, taking $\rho=C(\|g\|_{L^s(\Omega)},\|\Omega\|,\rho_0,p,c_3,c_4,c_5)$ and $T=T_2$.

We complete the proof.

Proposition 3.2. *The semigroup* $\{S(t)\}_{t\geq 0}$ *is norm-to-weak continuous on* $S(B_0)$, *where* B_0 *is the* $(L^2(\Omega),\mathcal{D}_0^{1,p}(\Omega,a)\cap L^q(\Omega))$ *- bounded absorbing set obtained in Proposition 3.1.*

Proof. Choosing $Y = L^2(\Omega)$, $X = \mathcal{D}_0^{1,p}(\Omega,a) \cap L^q(\Omega)$, the conclusion follows imme-diately from Theorem 3.2 in [22].

Theorem 3.1. *($(L^2(\Omega), L^2(\Omega))$- global attractor) Suppose that the hypotheses* **(H1)**,**(H2)** *and* **(H3)** *hold. Then the semigroup S(t) generated by problem (1.1) has an $(L^2(\Omega), L^2(\Omega))$- global attractor $\mathcal{A}_2$.*

Proof. We distinguish two cases to deal with our problem since it is subcritical if $\alpha \in (0, p + \frac{n(p-2)}{2})$, and supcritical if $\alpha \in [p + \frac{n(p-2)}{2}, n(p-1)]$.

Case 1. If $\alpha \in (0, p + \frac{n(p-2)}{2})$, then $p_\alpha^* \in [\frac{n}{n-1}, 2]$ and $\mathcal{D}_0^{1,p}(\Omega,a) \subset\subset L^2(\Omega)$.

Using the so-called uniformly compact method, the existence of the bounded abosorbing set in $\mathcal{D}_0^{1,p}(\Omega,a) \cap L^q(\Omega)$ yields the existence of a global attractor in $L^2(\Omega)$ immediately.

Case 2. If $\alpha \in [p + \frac{n(p-2)}{2}, n(p-1)]$, then $p_\alpha^* \in [\frac{n}{n-1}, 2]$ and $s \geq 2$. We infer from Lemma 2.1 that there exists some $r \in [1, p_\alpha^*)$ such that $\mathcal{D}_0^{1,p}(\Omega,a) \subset\subset L^r(\Omega)$\$.

Obviously, B_0 is also a closed bounded absorbing set in $\mathcal{D}_0^{1,p}(\Omega,a)$, we can consider our problem only in B_0 and B_0 has a finite ε- net in $L^r(\Omega)$.

Using the so-called uniformly compact method, we deduce that there exists a global attractor in $L^r(\Omega)$. From Corollary 5.7 in [22] and Proposition 3.1, we need only verify for any $\varepsilon > 0$ and any bounded subset $B \subset L^2(\Omega)$, there exists $T = T(\varepsilon, B)$ and $M = M(\varepsilon)$ such that $\varepsilon > 0$ and any bounded subset $B \subset L^2(\Omega)$, there exists $T = T(\varepsilon, B)$ and $M = M(\varepsilon)$ such that

$$\int_{\Omega(|u| \geq 2M)} |u(t)|^2 \, dx < C\varepsilon \quad \text{for } u_0 \in B, t \leq T$$

where the positive constant C is independent of ε and B. It follows from Lemma 5.2 in [22] that for any fixed $\varepsilon > 0$, there exist $\delta > 0, T = T(B)$ and $M = M(\varepsilon)$ such that the Lebesgue measure $|\Omega(|S(t)u_0| \geq M)| \leq \delta$ for all $u_0 \in B$ and $t \geq T$ and

$$\int_{\Omega(|S(t)u_0| \geq M)} |g|^2 < \varepsilon. \tag{3.12}$$

We multiply the first equation in (1.1) by $(u - M)_+$, one gets

$$u_t(u-M)_+ - div(a(x)|\nabla u|^{p-2} \nabla u)(u-M)_+ + f(u)(u-M)_+ = g(x)(u-M)_+$$

where $(u-M)_+$ denotes the positive part of $(u-M)$, that is,

$$(u\text{-}M)_+ = \begin{cases} u-M, & if \quad u \ge M, \\ 0, & if \quad u < M. \end{cases}$$

If M is a large enough positive constant, it follows from (1.2) that $f(u) \ge \tilde{c}|u|^{q-1}$ for all $u \ge M$.

Therefore, $f(u)(u-M)_+ \ge \tilde{c}|u|^{q-1}(u-M)_+ \ge \tilde{c}(u-M)_+^q.$ (3.13)

Hence,

$$\frac{1}{2}\frac{d}{dt}\|(u-M)_+\|^2_{L^2(\Omega)} + \int_{\Omega(u\ge M)} a(x)|\nabla u|^p dx + \int_{\Omega(u\ge M)} f(u)(u-M)_+ dx = \int_{\Omega(u\ge M)} g(u-M)_+ dx.$$

Since $L^q(\Omega) \subset L^2(\Omega), (q \ge 2)$, we deduce from Lemma 2.2, (3.12) and (3.13) that

$$\frac{d}{dt}\|(u-M)_+\|^2_{L^2(\Omega)} + C\|(u-M)_+\|^q_{L^2(\Omega)} \le C\varepsilon.$$

It follows from Lemma 3.2 in [4] that $\|(u-M)_+\|^2_{L^2(\Omega)} \le C\varepsilon$ (3.14)

for M, T large enough and $t \ge T$. Replacing $(u-M)_+$ by $(u+M)_-$,

we obtain also similar assertion $\|(u+M)_-\|^2_{L^2(\Omega)} \le C\varepsilon,$ (3.15)

where $(u\text{+}M)_- = \begin{cases} u+M, & if\ u \le -M, \\ 0, & if\ u > -M. \end{cases}$

It follows from (3.14) anh (3.15) that $\int_{\Omega(|u|\ge M)} (|u|-M)^2 dx < C\varepsilon$.

Therefore,

$$\int_{\Omega(|u|\ge 2M)} |u|^2\, dx = \int_{\Omega(|u|\ge 2M)} (|u|-M+M)^2 dx \le 4\int_{\Omega(|u|\ge 2M)} (|u|-M)^2 dx + 4\int_{\Omega(|u|\ge 2M)} M^2 dx < C\varepsilon,$$

for M large enough and C is independent of ε and B.

As a consequence, the semigroup $S(t)$ has an $(L^2(\Omega), L^2(\Omega))$- global attractor $\mathcal{A}_2$.

We now show that the existence of an $(L^2(\Omega), L^q(\Omega))$- global attractor.

Theorem 3.2. *($(L^2(\Omega), L^q(\Omega))$- global attractor) Assume that the hypotheses* **(H1)**, **(H2)** *and* **(H3)** *are satisfied. Then the semigroup $S(t)$ associated to (1.1) has an $(L^2(\Omega), L^q(\Omega))$- global attractor $\mathcal{A}_q$.*

Proof. Thanks to Corollary 5.7 in [22], Proposition 3.1 and Theorem 3.1, it is sufficient to prove that for any $\varepsilon > 0$ and any bounded subset $B \subset L^2(\Omega)$, there exist two positive constants $T = T(\varepsilon, B)$ and $M = M(\varepsilon)$ such that $\int_{\Omega(|u|\ge 2M)} |u|^q < C\varepsilon$, for all $u_0 \in B$ and $t \ge T$, where the constant C is independent of ε and B.

Obviously, we also deduce from Lemma 5.2 in [22] that for any fixed $\varepsilon > 0$, there exist $\delta > 0, T = T(B)$ and $M = M(\varepsilon)$ such that the Lebesgue measure $|\Omega(|S(t)u_0| \geq M)| \leq \delta$ for all $u_0 \in B$ and $t \geq T$ and $\int_{\Omega(|S(t)u_0| \geq M)} |g|^s < \varepsilon.$ (3.16)

We now multiply the first equation in (1.1) by $(u-M)_+^{q-1}$, for M large enough, one gets $u_t(u-M)_+^{q-1} - div(a(x)|\nabla u|^{p-2}\nabla u)(u-M)_+^{q-1} + f(u)(u-M)_+^{q-1} = g(x)(u-M)_+^{q-1}.$ (3.17)

Since $f(u) \geq \tilde{c}|u|^{q-1}$ with $u \geq M$. Thus,

$$f(u)(u-M)_+^{q-1} \geq \tilde{c}|u|^{q-1}(u-M)_+^{q-1} = \frac{\tilde{c}}{2}|u|^{q-1}(u-M)_+^{q-1} + \frac{\tilde{c}}{2}|u|^{q-1}(u-M)_+^{q-1} \tag{3.18}$$

$$\geq \frac{\tilde{c}}{2}|u|^{q-1-\frac{q-1}{s-1}}(u-M)_+^{(q-1)\frac{s}{s-1}} + \frac{\tilde{c}}{2}|u|^{q-2}(u-M)_+^{q} \geq \frac{\tilde{c}}{2}M^{q-1-\frac{q-1}{s-1}}(u-M)_+^{(q-1)\frac{s}{s-1}} + \frac{\tilde{c}}{2}M^{q-2}(u-M)_+^{q}.$$

We also have

$$g(u-M)_+^{q-1} \leq \frac{\tilde{c}}{2}M^{q-1-\frac{q-1}{s-1}}(u-M)_+^{(q-1)\frac{s}{s-1}} + C|g|^s. \tag{3.19}$$

Putting (3.18), (3.19) together with (3.17) and integrating over $\Omega(u \geq M)$, we obtain

$$\frac{1}{q}\frac{d}{dt}\int_{\Omega(u\geq M)}(u-M)_+^q dx + (q-1)\int_{\Omega(u\geq M)} a(x)|\nabla u|^p (u-M)_+^{q-2}dx + \frac{\tilde{c}}{2}M^{q-2}\int_{\Omega(u\geq M)}(u-M)_+^q dx \leq C\int_{\Omega(u\geq M)}|g|^s dx$$

and then $\dfrac{d}{dt}\int_{\Omega(u\geq M)}(u-M)_+^q dx + \dfrac{\tilde{c}}{2}qM^{q-2}\int_{\Omega(u\geq M)}(u-M)_+^q dx \leq Cq\int_{\Omega(u\geq M)}|g|^s dx.$

By the Gronwall inequality, the following estimate follows from taking M large enough and using (3.16), we have $\int_{\Omega(u\geq M)}(u-M)_+^q dx < \varepsilon,$ (3.20)
where C is independent of ε and M. Repeating the same steps above, just taking $(u+M)_-$ instead of $(u-M)_+$ we also obtain

$$\int_{\Omega(u\leq -M)}(u+M)_-^q dx < \varepsilon. \tag{3.21}$$

In both cases, we imply from (3.20) and (3.21) that $\int_{\Omega(|u|\geq M)}(|u|-M)^q dx < \varepsilon,$ (3.22)
for M large enough. Therefore,

$$\int_{\Omega(|u|\geq 2M)}|u|^q dx = \int_{\Omega(|u|\geq 2M)}|u-M+M|^q dx \leq 2^q\int_{\Omega(|u|\geq 2M)}(|u|-M)^q dx + 2^q\int_{\Omega(|u|\geq 2M)}M^q dx < C\varepsilon,$$

for M large enough. As a consequence, the semigroup $S(t)$ also has an $(L^2(\Omega), L^q(\Omega))$-global attractor $\mathcal{A}_q$.

It is possible to show that the regularity of the attractor increases. First, we will give a *priori* estimate for u_t endowed with L^2- norm.

Lemma 3.1. *Assume that the hypotheses* $(\mathbf{H1})$, $(\mathbf{H2})$ *and* $(\mathbf{H3})$ *hold. Then for any bounded subset* B *in* $L^2(\Omega)$, *there exists a positive constant* $T = T(B)$ *such that* $\| u_t(s) \|^2_{L^2(\Omega)} \le \rho_1$, *for all* $u_0 \in B$, *and* $s \ge T$, *where* $u_t(s) = \frac{d}{dt}(S(t)u_0)|_{t=s}$ *and* ρ_1 *is a positive constant independent of* B.

Proof. By differentiating the first equation in (1.1) in time and denoting $v = u_t$, we get

$$v_t - div(a(x)|\nabla u|^{p-2}\nabla v) - (p-2)div(a(x)|\nabla u|^{p-4}(\nabla u.\nabla v)\nabla u) + f'(u)v = 0. \tag{3.23}$$

Multiplying the above equality by v, integrating over Ω and using (1.3), we have

$$\frac{1}{2}\frac{d}{dt}\| v \|^2_{L^2(\Omega)} + \int_\Omega a(x)|\nabla u|^{p-2}|\nabla v|^2 dx + (p-2)\int_\Omega a(x)|\nabla u|^{p-4}(\nabla u.\nabla v)^2 dx \le c_3 \| v \|^2_{L^2(\Omega)}. \tag{3.24}$$

Hence $$\frac{d}{dt}\| v \|^2_{L^2(\Omega)} \le 2c_3 \| v \|^2_{L^2(\Omega)} \tag{3.25}$$

On the other hand, integrating (3.9) from t to $t+1$ and combining (3.10), we get

$$\int_t^{t+1} \| u_t \|^2_{L^2(\Omega)} dx \le C. \tag{3.26}$$

as t large enough. Combining (3.25) with (3.26) and using the uniform Gronwall inequality, we have $\| u_t \|^2_{L^2(\Omega)} \le \rho_1$, as t large enough, and ρ_1 is a some positive constant. The proof is complete.

For $(L^2(\Omega), \mathcal{D}_0^{1,p}(\Omega,a) \cap L^q(\Omega))$- global attractor, we need the following lemma ([5, Theorem 2.7])

Lemma 3.2. Let X be a Banach space and Z be a metric space. Let $\{S(t)\}_{t\ge 0}$ be a semigroup on X such that: (i) $\{S(t)\}_{t\ge 0}$ has an (X,Z)- bounded absorbing set B_0;

(ii) $\{S(t)\}_{t\ge 0}$ is (X,Z)- asymptotically compact;

(iii) $\{S(t)\}_{t\ge 0}$ is norm-to-weak continuous on $S(B_0)$.

Then $\{S(t)\}_{t\ge 0}$ has an (X,Z)- global attractor.

Theorem 3.3. Assume that the hypotheses $(\mathbf{H1})$, $(\mathbf{H2})$ and $(\mathbf{H3})$ are satisfied. Then the semigroup $\{S(t)\}_{t\ge 0}$ associated to (1.1) has an $(L^2(\Omega), \mathcal{D}_0^{1,p}(\Omega,a) \cap L^q(\Omega))$- global attractor $\mathcal{A}$.

Proof. By Lemma 3.2 and Propositions 3.1-3.2, we only need to show that the semigroup $\{S(t)\}_{t\ge 0}$ is $(L^2(\Omega), \mathcal{D}_0^{1,p}(\Omega,a) \cap L^q(\Omega))$- asymptotically compact.

This means that we take B a bounded subset of $L^2(\Omega)$, we will show that for any $\{u_{0n}\} \subset B$ and $t_n \to +\infty$, $\{u_n(t_n)\}_{n=1}^{\infty}$ is precompact in $\mathcal{D}_0^{1,p}(\Omega,a) \cap L^q(\Omega)$, where $u_n(t_n) = S(t_n)u_{0n}$.

By Theorem 3.2, it is sufficient to verify that $\{u_n(t_n)\}_{n=1}^{\infty}$ is precompact in $\mathcal{D}_0^{1,p}(\Omega,a)$. To do this, we will prove that $\{u_n(t_n)\}$ is a Cauchy sequence in $\mathcal{D}_0^{1,p}(\Omega,a)$. Thanks to Theorems 3.1-3.2, one can assume that $\{u_n(t_n)\}$ is a Cauchy sequence in $L^2(\Omega)$ and in $L^q(\Omega)$. Since $L_{p,a}$ is strongly monotone when $p \geq 2$. We have

$$\delta \| u_n(t_n)-u_m(t_m)\|^p_{\mathcal{D}_0^{1,p}(\Omega,a)} \leq \langle L_{p,a}u_n(t_n)-L_{p,a}u_m(t_m), u_n(t_n)-u_m(t_m)\rangle$$

$$= \langle -\frac{d}{dt}u_n(t_n) - f(u_n(t_n)) + \frac{d}{dt}u_m(t_m) + f(u_m(t_m)), u_n(t_n)-u_m(t_m)\rangle$$

$$\leq \int_\Omega \left|\frac{d}{dt}u_n(t_n) - \frac{d}{dt}u_m(t_m)\right| |u_n(t_n)-u_m(t_m)|\, dx$$

$$+ \int_\Omega |f(u_n(t_n)) - f(u_m(t_m))| |u_n(t_n)-u_m(t_m)|\, dx$$

$$\leq \left\| \frac{d}{dt}u_n(t_n) - \frac{d}{dt}u_m(t_m)\right\|_{L^2(\Omega)} \| u_n(t_n)-u_m(t_m)\|_{L^2(\Omega)}$$

$$+ \| f(u_n(t_n)) - f(u_m(t_m))\|_{L^{q'}(\Omega)} \| u_n(t_n)-u_m(t_m)\|_{L^q(\Omega)}$$

Because $f(u_n(t))$ is bounded in $L^{q'}(\Omega)$, in addition, from Lemma 3.1, we deduce that $\{u_n(t_n)\}$ is the Cauchy sequence in $\mathcal{D}_0^{1,p}(\Omega,a)$.

4. Conclusion

In this paper, we study an initial boundary value problem for a class of quasilinear degenerate parabolic equations involving weighted p-Laplacian operators by using the compactness method and weak convergence techniques in Orlicz spaces. The long-time behavior of solutions to the problem is considered via the concepts of global attractors in various bi-spaces.

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SMART AND SAFETY AC POWER SOCKET FOR AN IOT APPLICATION OF THE SMART HOME

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Received: 22 February 2019/ Accepted: 11 June 2019/ Published: June 2019

Abstract: *This paper presents a safety power socket system that can be controlled wirelessly and that has been specifically designed to monitor electrical events. Each socket of the system embeds a microcontroller, 2.4 GHz Wifi chip, relays, and current sensor. The main features of the system include the remote control of the power outlet, real-time monitoring of the current consumption, the customization and programming of the power supply time schedule, the automatic interruption of vampire currents, and the prevention of certain types of electrical fires and electrocutions. A prototype with such features has been implemented and tested in a simple home automation network in order to validate its functionalities. The results show that the system reacts fast and avoids overconsumptions and electrocutions, being able to make the next generation of homes safer and smarter.*

Keywords: *IoT, Smart socket, safety socket, smart home.*

1. Introduction

Smart home is recently rising as one of the best smart IoT application. In the modern homes, the electricity is provided to electrical devices through the last element of the power supply chain, the power sockets, which have been regarded traditionally as a mere junction. In fact, power sockets have not evolved as fast as other everyday devices, although they seem to be one of the best positioned candidates to be improved as smart homes are becoming increasingly popular.

Recently, different functionalities have been added to power sockets, like wireless control or current consumption monitoring, but there are still many others that could make such devices safer and smarter. This paper addresses some of the most common problems that arise when interacting with power sockets and shows a novel approach to two of them: the prevention of electrical fires and the avoidance of electrical shocks.

Regarding electrical fires, they are described as fires that begin because of some type of electrical failure or malfunc- tion. The National Fire Protection Association (NFPA) [1] latest report states that, in 2011, just in the USA, there were 47,700 home electrical fires that resulted in 418 civilian deaths, 1,570 civilian injuries, and $1.4 billion in direct property damage [2]. In the same report, it is also stated that, of all the electrical fires whose cause could be determined precisely, the vast majority were caused by short-circuits.

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A similar situation occurs with electrical shocks. According to the Electrical Safety Foundation [3], in North America each day nearly 7 children are treated in hospital emergency rooms for electric shock or burn injuries caused by tampering with a wall outlet. In the same region, fires and burns are the third leading cause of unintentional death among children aged 14 and under.

In order to address the two issues previously mentioned, this paper presents a smart power socket system that has the following features. It is able to detect overconsumptions that might lead to overheat in low-current systems and, therefore, to an electrical fire. This device prevents electrocutions, since it only supplies power when it identifies a valid appliance. The smart socket can be controlled remotely using Wifi technology. It is also possible to monitor in real-time and make available to external devices (e.g., PCs connected to the Internet, smartphones, and tablets) current consumption data. The intelligent outlet can disconnect the power supply when a vampire current is detected (a vampire current is a current that arises when an appliance consumes power when it is in stand-by mode or when it claims to be switched off).

2. Design of the Socket

2.1. The mechnical structure of the socket

The socket is designed including three main parts. First, the tempered glass is used for save, durable and sensitive touching button. It is also very pretty for good design of user interface. Second, this is a core of the smart socket which is PCB circuit with IC and microcontroller mounted on its surface. The program is installed into the microcontroller to perform functions that socket has to possess. The last component is a box to cover and fix the PCB circuit which is made from ABS plastic. The design model of the socket is shown in the Fingure 1.

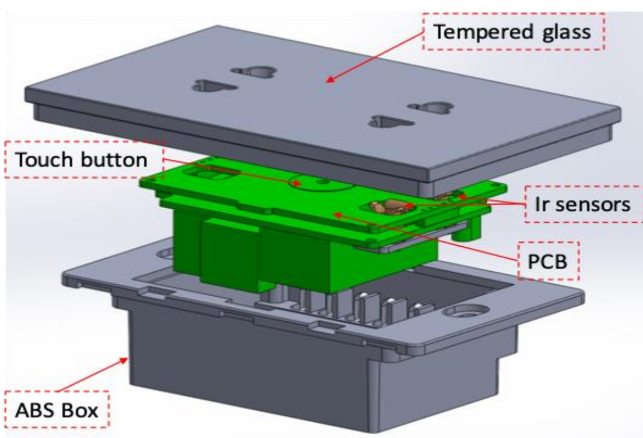


Figure 1. *The mechanical design of the socket*

2.2. Designing the prevention of electrical fires

Electrical fires can occur due to different problems, with the most common ones being power overloads, short-circuits and arcs. In cases, when they happen, the energy released must be smaller than the maximum load capacity of the transmission medium to avoid irreparable damage.

Figure 2. *Arc causing fire on the socket*

In the case of power overloads, the current must remain below the maximum permissible load current of the transmission medium (I_z). The following rule is commonly applied in accordance to IEC/EN 60898-1/-2 and IEC/EN 60947-2 by miniature circuit breakers (MCBs) manufacturers to determine when the circuit should be opened in the event of an overload:

$$I_{t=30} \leq 1.45 I_z \quad (1)$$

where $I_{t=30}$ is the monitored current at an ambient temperature of 30˚C.

In the case of short-circuits, it is necessary to determine the maximum short-circuit current. If the short-circuit release time is under 100 ms, the next rule is usually followed to prevent the devices from firing and avoid damage [4]:

$$K^2 S^2 \geq I_k^2 t_{cut-off} \quad (2)$$

where K is coefficient of the material medium (As/mm2), S is cross-sectional area of the conductor of the transmission medium (in mm2), I_k is RMS value of the short-circuit current flowing through the medium (in A), and $t_{cut-off}$ is release time in seconds of the protection device for the current. Equation (2) indicates that if a high current is maintained during too much time, the conductor through which it flows might be melted and even catch on fire. Such equation in (2) is derived from thermodynamic equations that regulate adiabatic processes and it enables designers to check the suitability of a composite cable used for the c.p.c. (circuit protection conductor, i.e., the earthing conductor and all the equipotential bonding conductors).

The case of arc causing fire can be avoided by preventing the generation of arc. Generally, the arc occurs because the pins of plugs are close to terminals of the socket under high voltage between them as shown in figure 2. The arc cause of emitting arc reveals a mechanism for contacting and transmitting current in the socket without arc. It is the plug's pins mounted in socket's terminals with zero voltage in both sides. To achieve this mechanism, this research proposes a method which is the electricity being supplied to the socket's teminals after 3 second when the plug's pins were plugged in the socket.

2.3. Designing the prevention of electrical shocks

Power socket usually includes different safety measures in order to prevent electrical shocks. The most common one is the addition of a third prong (ground) that reduces the risk of electric shock and protects equipment from electrical damage. There are also power sockets that are encapsulated into a tamper resistant receptacle that prevents the insertion of objects. There are also power sockets (AFCI, Arc Fault Circuit Interrupters) that reduce the risk of electrical shock by interrupting power when arc fault occurs in the circuit. Moreover, there is another kind of sockets (GFCI, Ground Fault Circuit Interrupters) that prevents shocks by shutting off the power when the electricity flowing into the circuit is superior to the one returning (i.e., when there is a leakage). All these traditional solutions work in most situations, especially when the same outlet combines different functionalities (e.g., ground fault protection and tamper resistance), but there are still cases when they fail (e.g., when the ground is not properly set or when a kid is able to tamper the socket).

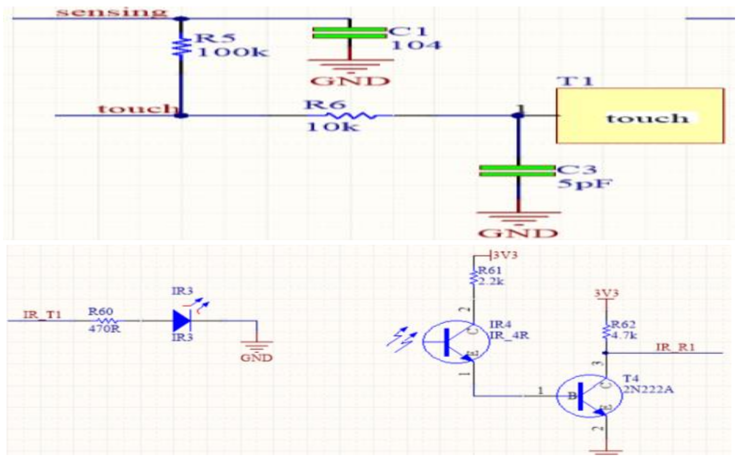


Figure 3. *Schematic circuit of touch and infrared sensor*

The system proposed in this paper prevents electrical shocks based on the following principle: if there is no appliance connected to the outlet, electricity will not be supplied. In this way, most of the tampering will have no consequences and no electrical shocks should occur. To achieve this goal, it is necessary to identify when an appliance is connected to an outlet. There are different mechanisms that allow for identifying appliances, but most of them rely on mechanical systems, nonautomated systems (e.g., pressing a button to indicate that the appliance is connected), or proprietary technologies. This design uses infrared sensors to detect plug's pins in the socket's terminals for automation mode and using a touch button for nonautomated mode. Figure 3 shows the button and infrared sensors mounted on PCB circuit in the sockets.

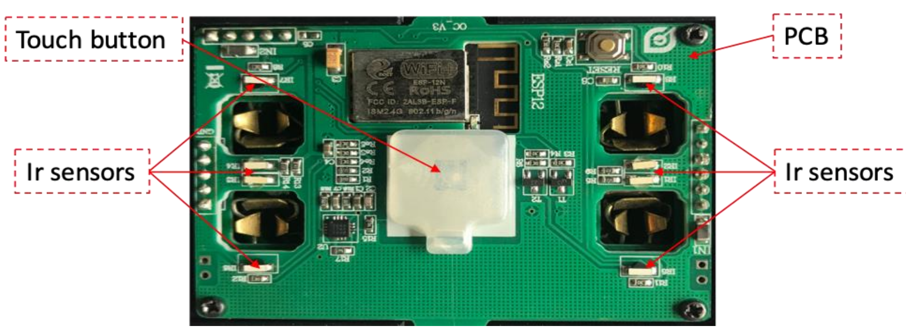


Figure 4. *Implementation of touch and infrared sensors*

2.4. Designing prevention of a vampire current

The vampire current is a significant problem to home energy efficiency. Surprisingly, just a few smart power outlets [5] automate the detection of vampire currents and are able to disconnect appliances after they have been in stand-by for a certain amount of time. If the current exceeds the maximum allowed during $t_{cut\text{-}off}$ (duration to open circuit), the system turns off the power supply to avoid damage and electrical fires. In order to perform this function, we design the socket hardware to measure power consumtion of appliances including voltage and current measurement.

2.4.1. Background math for measurement

In an AC circuit voltage, current, and power are defined as follows:

$$v(t) = V.\cos(wt) \tag{3}$$

$$i(t) = l.\cos(wt - \theta) \tag{4}$$

$$p(t) = v(t).i(t) = V.l.\cos(wt).\cos(wt - \theta) \tag{5}$$

When the load is purely resistive, voltage and current are in phase. When the load is either inductive or capacitive, voltage and current are out of phase. Using several trigonometric identities, the power can be expressed as:

$$p(t) = \frac{v.i}{2}.\cos(\theta).[1 + \cos(2wt)] + \frac{v.i}{2}.\sin(\theta).\sin(2wt) \tag{6}$$

The average (real) power, P. Real power is the energy that flows to the load. It is what the electric company bills home users for. It can then be written as:

$$P = V_{mrs}.l_{mrs}.\cos(\theta) \tag{7}$$

Reactive power, Q, is the energy that flows back and forth in an inductive or capacitive load. On average, no reactive power is consumed. It can be written as:

$$Q = V_{rms}.l_{rms}.\sin(\theta) \tag{8}$$

Together, real power and reactive power form complex power. This is the actual power that the electric company is supplying. It can be written as:

$$S = P + jQ \tag{9}$$

The magnitude of complex power is called apparent power, |S|, in units Volts-Ampere. The power factor, PF, a measure of efficiency. It is defined as:

$$PF = \cos(\theta) = \frac{P}{|S|} \tag{10}$$

2.4.2. Voltage measurement

To measure voltage, the general idea is to use a very large voltage divider to divide the 270 V peak-to-peak signal down to level which can be sampled by the ADC. Using a 1001:1 voltage divider (with 1 MΩ and 1 kΩ), 270 V peak-to-peak is divided down to 0.27 V peak-to-peak. A very large resistor (1 MΩ) was used in the divider to limit the current between AC live and neutral. Assuming a 270 V drop, only 0.27 mA flows through the 1 MΩ resistor, dissipating 0.03 W, well within the power ratings of the resistor. To calculate the line voltage from the voltage divider output, the following equation can be used:

$$V_{line} = 1001 \times V_{divider} \quad (11)$$

2.4.3. Current measurement

To measure current, the general idea is to break the neutral line and insert a very small current-sensing resistor (0.2 Ω). This would create a small voltage difference across the resistor. Since we know the voltage drop and the resistor value, we can mathematically determine the current through the neutral line. Since the resistance is very small, very little power is dissipated through it. We carefully checked the ratings of this and other circuit components for power ratings. The resistor is rated for 3 W. We expected a 2000 W applician to draw about 10 A. This results in a voltage drop of 0.54 V and a power dissipation of 0.58 W. To calculate the line current from the current-sensing resistor voltage drop, the following equation can be used:

$$I_{line} = \frac{V_{current-sense}}{R_{current-sense}} = \frac{V_{current-sense}}{0.2\Omega} \quad (12)$$

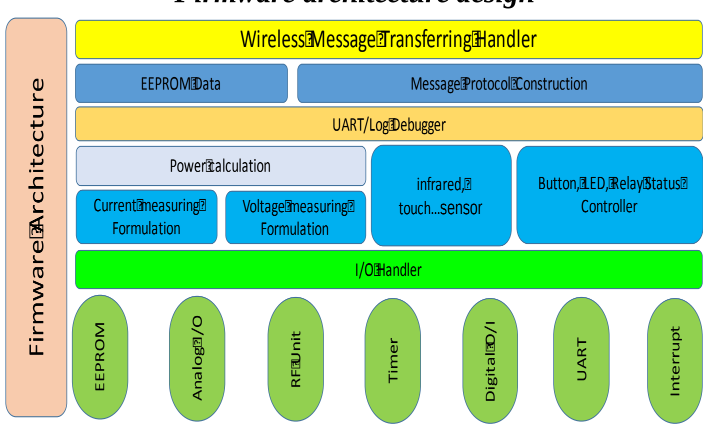
2.4.4. Power calculations

To calculate real power, voltage and current are multiplied during every sample and summed up. After 1000 samples are taken, the power summation is divided by 1000 to get the average power over 1 second. The following equation describes the operation:

$$P = \sum_{i=1}^{1000} \frac{IV}{1000} \quad (13)$$

To calculate apparent power, V_{rms} and I_{rms} should be multiplied together every second. Power factor was calculated by dividing average power from apparent power.

Firmware architecture design



***Fingure 5.** Software architeture*

As shown in the figure, the software is designed to handle every signal come in and out from a microcontroller to perform the socket's functions. Physical signals from input and output pin of the microcontroler is processed by a software module called I/O hanlder. It encodes raw data then transfer them to a higher processing modules such as: current, voltage formulator, sensing data processor and relay, display controller. Needed data to perform tasks is extracted at these modules and transfered to a wifi processor via the UART protocol.

The used wifi processor, Esp8266 is a low-cost Wi-Fi microchip with full TCP/IP stack and microcontroller capability. It is capable to program the proposed functions of the sockets. The api (Application programming interface) of the sockets function is implemented to be able to control and monitor from a third-party smartphone application such as turning on/off, timing turn on/off, sending consuming power,etc. The socket becomes an IoT device because these apis make it can be controlled and monitored via the internet. Therefore, users can manage their household appliances easily from smartphones.

3. Experimental Analysis

3.1. Experiment setup

The block diagram is shown in Figure 6. Our device is placed between the mains line and the device under test. Current measurements occur on the neutral line. The relay is switched on the AC live line. A voltage divider is used to step down the voltage to the correct levels. All signals are optically isolated before reaching the MCU.

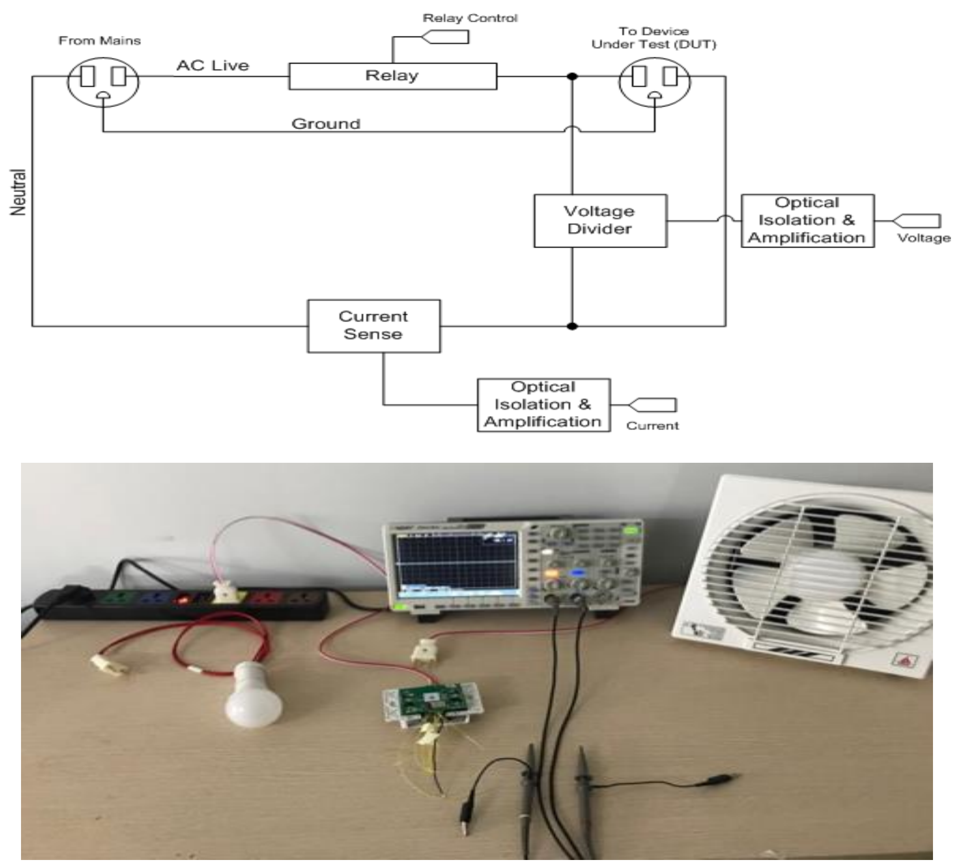


Figure 6. *Experiment setup*

3.2. Appliance detection

In order to detect an appliance plugging into the socket's terminals, the infrared sensors circuit is implemented as shown in the figure. The output pins of sensor circuit are connected to an oscilloscope XDS3102A to fingure out its response. Once the plug's pins plug into the socket, the infrared sensor circuits response as signals of high level. The circuit returns to a signal of low level if the plug is drawn out from socket. The experiment results are shown in the figure 7.

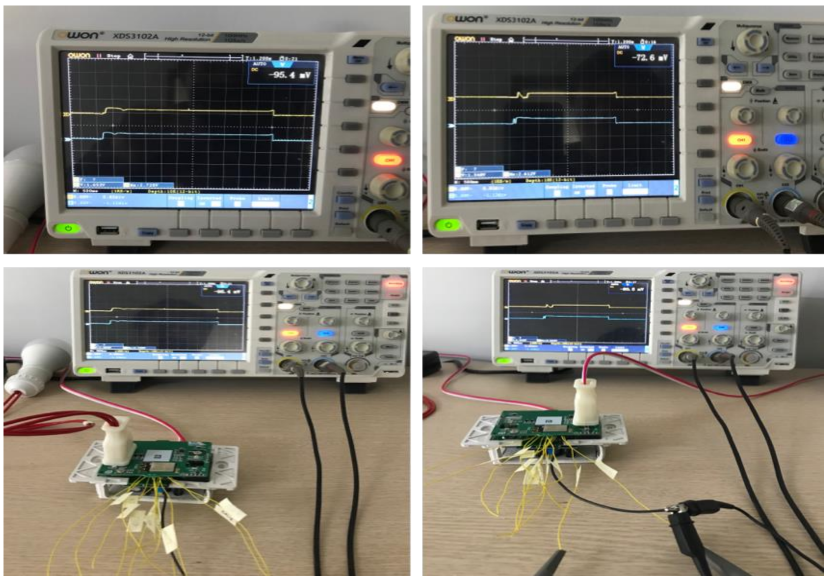


Figure 7. *Deteced Signals of plugged appliances*

3.3. Current and Voltage Monitoring

We designed this device to measure stable devices, so the 1 Hz report time is not terribly slow as there are very few devices that have power draw that changes faster than once a second. The designed application for this device is for power monitoring over duration of time, as it also keeps track of total energy used through the plugs. Also, the commercial power measuring device that we have also only updates at 1 Hz.

The error in measurement can be from opto-isolator noise as well as the lack of precision in ADC and data calculation. To test the accuracy of our project, we used a commercial power measuring device to measure the statistics of power drawn by a single 50W light bulb. The results are organized in Table 1.

Table 1. *Accuracy Comparison for a 50 Watt Light Bulb*

	Our Project	Commercial Product	Error
Total Power Draw (W)	51.9	50.6	2.57%
RMS Voltage (V)	120.0	121.5	1.23%
RMS Current (A)	0.43	0.409	5.13%

The absolute difference in the power measurement is about 1W, which we consider pretty good. We feel that with higher load or even a calibration our device can achieve better accuracy. According to this result, any vampire current will be detected and household appliances will be insolated for protection functions.

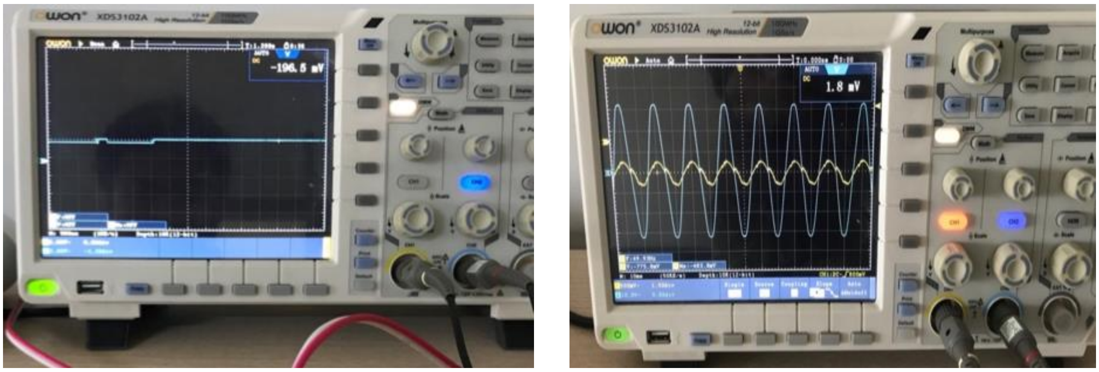


Figure 8. *Voltage and current measurement*

4. Conclusion

Our final design of the device mostly met our original goals. We not only planned to be able to monitor and control the device remotely by wifi protocol, but also the socket can be detected with risks of shock, fire, vampire current, etc. to isolate househole appliances or human from electricity network. We decided to finish a working version of the project on a timely basis and implement other features such as power factor and the power based auto shutoff. In conclusion, the designed socket has good performance of safety functions.

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APPLYING THE BUTLER'S TOURIST AREA LIFE CYCLE MODEL TO CLASSIFY TOURIST DESTINATIONS ON THANHHOA-NGHEAN'S COASTLINE

Le Van Truong

Received: 3 October 2018 / Accepted: 11 June 2019 / Published: June 2019

Abstract: *The author applied Butler's model of the life cycle of Tourst destinations, score-based measures and visitor satisfaction surveys to classify tourism destinations along 184 km of Thanh Hoa - Nghe An's coastline. The results of a total of 32 tourism destinations were evaluated and classified in this region with 21 sites, during the exploration phase, 9 sites at the involvement stage and only 2 sites, is Sam Son and Cua Lo, reached the development stage. The study also proposed solutions to 21 points in the exploration stage to other stages and 9 points in the phase to participate in the development phase within 5-10 years.*

Keywords: *Tourist Destination, Thanh Hoa - Nghe An Coastline, Classification, R.W Butler.*

1. Introduction

Based on the theory of "Stages of Economic Growth" by W. W Rostow (1960) [14] and "Product Life Cycle" by Raymond Vernon (1966), Applied in the field of tourism, in 1980, R. W Buttler developed the Tourist Area Life Cycle (TALC) theory [3]. From that point up, many studies have applied this theory to evaluate and classify tourism destinations in different territories of the world in order to find solutions to manage and promote tourist destinations [1], [4], [7], [9] ... However, such studies in Vietnam, Thanhhoa and Nghean are quite rare, only few works master theses [11], [12] ...

In December, 2016, the Ministry of Culture, Sports and Tourism issued a set of criteria for tourism destination assessment, but this set of criteria only applies to "The number of tourist arrivals is expected to reach 500,000 tourist arrivals per year .... and tourism destinations have been invested to develop tourism, attracting more than 50,000 tourists a year. "[10]. Tourist destinations with smaller numbers of visitors are not classified. In fact, that is the motivation for our study to classify the tourism destinations along the coast of Thanh Hoa-Nghe An on Butler's theory - a theory that has great scientific and practical significance.

2. Scope

All tourist destinations along 184 km of Thanh-Nghe coastline, including the tourist destinations have met the criteria by the Ministry of Culture, Sports and Tourism (currently the two provinces have not yet evaluated), already planned and not planned tourist destinations.

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3. The concept of Butler's tourism area cycle life

According to Butler’s TALC model a tourism destination goes through six stages.

The first stage is exploration. This stage involves a few tourists discovering a new tourism area (Butler, 2011). This stage is usually characterized with minimal number of visitors due to limited access, limited knowledge and inadequate facilities. Tourists visiting such areas are mainly the allocentric or adventure seeking tourists.

The second stage, involvement, begins when local community starts to participate in the tourism development process. On seeing a few tourists interested with their area, members of the local community begin to develop simple infrastructures and facilities such as access roads and small accommodation and catering facilities (Butler, 2011). More tourists gain knowledge of the area and the number of visitors begin to rise. At this stage the area is still popular among the allocentric type of tourists.

The development stage begins to set in when the government and small scale investors take notice of the interest of tourist on this area (Butler, 2011). The government set in to provide necessary infrastructure such as roads and electricity while investors begin to provide sophisticate infrastructures such as accommodation and transport facilities. The areas also become widely marketed as tourism destination further increasing the number of tourist visiting the area. The mid-centric tourists begin to have confident with this area and start visiting further increasing the number of visitation.

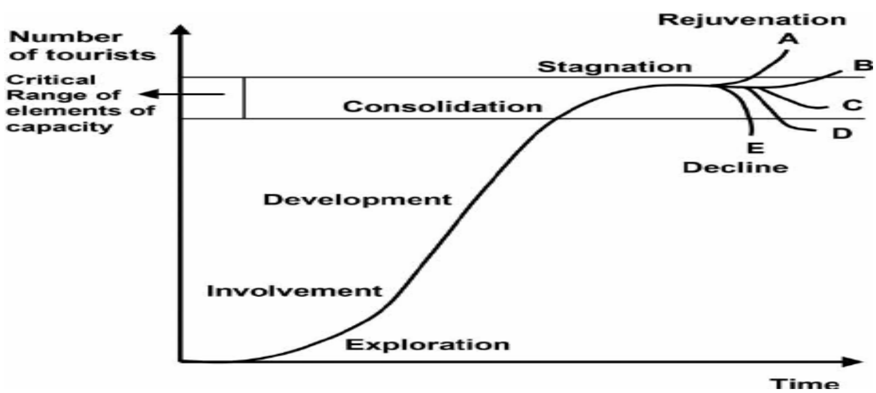

Diagram 1. *The theoretical life cycle of a destination [4]*

Consolidation stage is marked by the entry of the big players in the tourism industry into this area (Butler, 2011). Small facilities are replaced by large tourism establishments design to serve large masses of tourist. More and more tourist flock into the areas as the pyschocentric begins to gain confident with this destination.

The stagnation stage is marked by stabilization in the growth of the number of tourist (Butler, 2011). This is mainly as result of the allocentric tourist leaving the area as it begins to lose its novelty status and their number is replaced by the pyschocentric tourists who are appealed by the comfort and familiarity offered by the area. The rising number of tourists begins to irritate locals as they begin to feel the pressure associated with the large number of tourists. There are no further tourism developments as the area reaches its carrying capacity (Lagiewski, 2004).

Decline or rejuvenation stage may follow the stagnation stage. Decline stage results from the tourism areas undergoing degradation as result of increased pressure from the large number of tourists. Environmental and social problems begin to haunt the area leading to the area losing its appeal. However, the areas tourism appeal may be rejuvenated through various means such as developing new products beside the product that had initially attracted tourist into the area.

4. Methods

In order to reflect the complexity of the development of the Tourist Destination, the study used seven groups of criteria and specified 32 criteria. Of these, 31 criteria (total 85 points) were assessed by experts and 1 by criteria (15 points). Compared with the criteria set by the Ministry of Culture, Sports and Tourism, we added criteria for the number of visitors to the destination (30 points) with both domestic and foreign visitors. Therefore, the structure of each group of criteria, criteria (Table 1) is changed. For the satisfaction of visitors, we use the National Tourism Administration's questionnaire [10] and survey each site to 20-50 visitors.

Table 1. *Criteria for Tourism Destination assessment*

Criteria group	Criteria	Describe criteria	Score achieved/ Total score
Tourist Resources (15 p)	The diversity and uniqueness of the resource	There are beautiful or phenomenal scenery, special relics, including cultural buildings, historical monuments recognized national specialties or world heritage, national landscape, conservation garden country/ biosphere reserve/ world natural heritage; or can develop more tourism activities / products.	/10
	Protect and enhance resources	The original, well protected, beautiful scenery, still retain the traditional culture, take measures to protect, create attractive attractions separately.	/2
	Capacity	At least 140 visitor/day	/3
Visitors (30 p)	Domestic visitor	*Point*: 1 2 3 4 5 …………… 24 25 visitor /day: 0 5 10 15 20 25 ………… … 115 120 125	/25
	Foreign visitor	*Point:* 1 2 3 4 5 visitor /day: 0 3 6 9 12 15	/5

Products and services (17 p)	Provide information to visitor	Providing information for visitors via telephone 24/24, through website or social network, website, social network posting animations, videos, support download electronic publications of Tourism Destination, have FAQ., answers to frequently asked questions of visitors, online support, online contact email address, links to other related websites such as: travel agents, carriers ...; There is an informational brochure for visitors, a well-designed publication, an impressive image, unique, the right size to carry, complete information, easy to read, use friendly material. MT; Information in printed publications, electronic publications, websites or social networks is expressed by Vietnamese and English.	/1
	Information guide throughout the Tourist Destination	Having the rules of the entire Tourist Destination and detailed rules table in the functional areas; A map of the entire Tourist Destination and functional areas are located in many places; Having signage system leading to functional areas; Signboard in Vietnamese and at least 1 foreign language, designed with many images, symbols showing high information and information map location position of visitors.	/1
	Notes	Having tables of information on the subjects or having guest speakers; Having electronic information boards explaining to the visitors; Interpreters are able to use foreign languages to explain to guests/Having automatic interpretation services; There is support for hearing impaired guests.	/1
	Tourist information center	There is an information room with internet access for visitors looking for information about the DBDL; There are on-site staff providing information on receiving and settling complaints from tourists, ... There are additional information booths in the functional areas in the DDSDL (display publications advertising the Tourist Destination for visitors, prices for information publication is designed compact, convenient for the publication...).	/1
	Facilities for accommodation	Accommodation facilities of 3 star level or higher.	/2

	Services to guests in the residences	Varied services and preferential policies for visitors; No time limit for service; Employees have good skills, friendly, enthusiastic; Equal treatment for all customers; Dispense service prices.	/1
	Restaurant system	There are standard restaurants serving tourists, capable of serving 500 guests or more, are classified into restaurants, Asia, ...; Restaurant system is distributed to guests in the hotel and in functional areas.	/2
	Food Service	Ensure food safety ; The menu is diversified and there are preferential policies for customers; Flexible service time; Qualified staff, good skills, friendly, enthusiastic; Equal treatment for all customers; Dispense service prices.	/1
	Entertainment facilities	There is an amusement park with a variety of games such as outdoor amusement park, indoor playground, cinema, etc. for adults and children; Certificate of IAAPA.	/1
	Entertainment services	Ensure absolute safety for tourists; Varied services and preferential policies for customers; Qualified staff, good skills, friendly, enthusiastic; Equal treatment for all customers; Equipment for recreational activities to ensure standards and standards of manufacturers; Dispense service prices.	/0,5
	Performing arts activities	There are demonstration activities, art performances serving guests with frequency ≥ 1 time/day.	/0,5
	Services to visit, explore, learn the value of nature, culture	Ensure absolute safety for tourists; Employees have a wide knowledge, depth of the object, explore at the destination, good skills, friendly, enthusiastic; The tour program is rich, diverse and flexible; There is formal information about the participants; Equal treatment for all customers; Dispense service prices.	/3
	Event, conferences, seminars services	Variety of services and incentives for customers; Flexible service time; Qualified, good skills, friendly, enthusiastic staff; Modern equipment, rich fit the nature of various types of events; Equal treatment for all customers; Dispense service prices.	/1
	Shopping service	There is a shopping center for shopping needs of visitors or a shopping center with a registered standard for tourists; There are a variety of services catering to the needs of the guests (shopping, beauty, fashion consultancy, health consulting, personal	/1

		equipment hire, ...); Flexible service time; Qualified staff, good skills, friendly, enthusiastic, responsible; Make public service charges; Treat all customers equally.	
Management (8 p)	Management	Having a management board with clear organizational structure, functions and tasks; The Management Board shall handle the problems of tourist destinations in a smooth and responsible manner, ensuring the annual growth of the whole area; Issue and control the implementation of regulations and rules of behavior related to tourism related phenomena.	/1
	Natural environment and sanitation	Fresh air, no pollution; Surface water (lakes, ponds, rivers, streams, fountains, ...); Waste is not discarded along roads, tourist sites and surface water sources; Equipment in tourist destinations such as lights, signs, monuments, small scenes, clean and clean; The areas selling products, souvenirs, food items must ensure common sanitation; Waste is collected in the fixed locations; The area under construction is covered.	/1
	Process rubbish	Having a plan to ensure environmental sanitation in tourist destinations; Having a system of waste collection, average of at least 01 garbage bin with cover / 300m along internal roads; There is a place where garbage is collected from both destinations; Having a separate waste treatment system in the tourist destination or a means of transporting garbage to local waste treatment sites with a frequency of once a day; Use of specialized high tech support tools (using chemicals allowed for pollution treatment, waste treatment, etc.) in appropriate areas.	/1
	Sanitation	There are standard public toilets for tourists at the functional areas and main attractions; There are mobile toilets in other public areas.	/1
	The friendliness of the local community	Local communities do not engage in harassing tourists (clinging to customers for sale, offering services ...); Communicating, behaving cultural identity and civilization of local people; Ready to support tourists (directions, directions, ...); Respecting culture, behavior of tourists from the region to the local; Be ready to introduce and guide tourists to integrate and experience the local culture style.	/1
	Organization of security and order	There is a security and order department, specialized teams are located at points	/1

		and there is a team of supervisors, specialized patrols.	
	A plan to ensure security and safety for tourists	Having a plan to ensure security and safety for travelers in normal conditions with simple, common problems; There is a hot line connected to local departments to coordinate the work of ensuring security and safety for tourists. (Support the resort in coordinating the problem solving over functions. of the resort), dispatch staff on the hotline 24/7; To take the initiative in evacuating people and property in case of natural calamities, fires and other serious incidents.	/1
	Facilities to ensure security and safety for tourists	Have security checkpoints in the tourist destination and at each functional subdivision; Security forces are equipped with uniforms and the minimum equipment to ensure security and safety for tourists; Modern, high-resolution, high-resolution camera system for surveillance of major tourist attractions and functional areas; Automatic fire alarm system in the buildings; Having a loudspeaker system for reporting incidents and emergencies; Have a system of barriers to isolate dangerous areas; Water fountains and water tanks for firefighting in functional areas; There are special vehicles for patrol and security control in the tourist destination .. and specialized equipment for transportation or rescue such as bicycles, motorbikes, lifeboats.	/1
Infrastructure (5 p)	Road system	Access to the site can be directly connected to the national transportation system through at least 2 of the 4 modes of transport: road, rail, water, air (not through the system). inter-district roads or short distances); There is a gateway to the resort; These gateways are distributed from many directions, many different locations bordering the tourist area to facilitate the means of transportation.	/1
	Signs approaching tourist destinations	Signage signs, access to the resort; Placement of suitable signs (in front of crossroads); The number and location of the appropriate instructions (with signs at all forks, crossroads leading to the resort); Clearly designed signboards (size of signboards, font size, information picture, color, etc.), informative content; Bilingual expressions.	/1
	Internal roads	100% of internal road system is covered with concrete / plastic and wide to ensure traffic safety for 2 lanes.	/1
	Power system	Design and installation of lighting systems with modern, environmentally friendly equipment, energy saving along internal roads, accommodation, functional areas and all attractions. in the resort; Having backup power system; There is an art lighting system.	

	Water supply and drainage system	Having clean water system to meet the demand of clean water; Water purification system according to international standards (can be taken immediately without boiling) to serve the needs of guests; water and pipe systems, fountains for fire fighting; drainage system to ensure environmental hygiene; wastewater treatment system to ensure environmental safety; Waste water treatment for reuse for needs such as watering plants, sanitation, etc.	/1
Local participation (10 p)	Workers are local people in tourist destinations	Employers are local people (from 3% of total workers in the whole resort or more); Deductions from the turnover of the tourist area to support the annual movement of the locality; To deduct the turnover of the tourist areas in support of the construction and upgrading of local public works annually; The percentage of local households participating in doing business in the tourist area is 5% or more.	/10
Visitor Satisfaction (15 p) (Vouchers from Vietnam Tourism Administration)			/15

The total number of points per Tourist Destination in the range of 1 to 100 and is grouped into five groups that are characteristic of the four categories that follow the development of Butler's TALC model. Tourist Destinations have a score of 1.0 - 24.9, which is classified as the discovered stages (exploration); Scores 25.0 - 49.9 are classified as periods of involvement stagae; Scores 50.0 - 74.9 are classified as developmental stage; Scores 75.0 - 100,0 are classified as consolidation stage.

5. Classification results

The results of our study are summarized in Table 2.

According to the criteria tourism resources, Sam Son and Cua the highest scores (11/15 and 11/15 respectively). Followed by Hai Tien, Hai Hoa, Quynh Phuong, Quynh Bang, Cua Hien and Bai Lu reached 5/15.

According to the number of visitors, Sam Son is the best is followed by Cua Lo. These are also two destinations that have been developed for decades. The ones with fewer visitors are Hai Tien, Hai Hoa, Bai Lu, Cua Hien and Cua Hoi.

From the criteria for tourism products and services, Sam Son and Cua Lo reached the highest score (15/17). Very few tourist products, or local products that have not yet become commodities such as mangroves, are new discoveries.

Up to 21/32 destinations without management board and management are spontaneous (managed by individuals or commune level). This is also the point where infrastructure is almost undeveloped, especially infrastructure for tourism purposes. It is noteworthy that the unmanaged areas and undeveloped infrastructure have quite active local participation in tourism activities. Of course, these activities are not managed.

Table 2. *Summary of scores for each criterion group and classification of the Tourist Destinations by stage*

No.	Score of eachcriterion group / Tourist Destinations	1 (15)	2 (30)	3 (17)	4 (8)	5 (5)	6 (10)	7 (15)	Total	Stages
	Thanh Hoa province									
1	Ngason Mangroves	3	1	2	0	0	1	2	**9**	exploration
2	Daloc Mangroves, Hauloc	3	1	1	0	0	1	2	**8**	exploration
3	Haitien Seaside, Hoanghoa	5	5	5	4	2	3	6	**30**	involvement
4	Samson city	11	26	15	6	3	7	10	**78**	development
5	The South Samson Seaside	5	6	5	3	1	2	3	**25**	involvement
6	Quanghai Seaside, Quangxuong	3	2	2	0	0	1	1	**9**	exploration
7	Quangthai Seaside, Quangxuong	3	2	2	0	0	1	1	**9**	exploration
8	Quangloi Seaside, Quangxuong	5	6	4	3	2	2	4	**26**	involvement
9	Quangnham Seaside, Quangxuong	4	2	2	1	0	1	1	**11**	exploration
10	Haininh Seaside, Tinhgia	4	2	2	0	0	1	1	**10**	exploration
11	Ninhhai Seaside, Tinhgia	4	2	2	0	0	1	1	**10**	exploration
12	Haian Seaside, Tinhgia	4	2	2	0	0	1	1	**10**	exploration
13	Hailinh Seaside, Tinhgia	4	2	2	0	0	1	1	**10**	exploration
14	Tandan Seaside, Tinhgia	4	4	2	0	0	1	1	**11**	exploration
15	Haihoa, Tinhgia	5	9	6	2	2	2	4	**30**	involvement
16	Haithanh Seaside, Tinhgia	4	4	3	0	0	1	1	**13**	exploration
16	Haibinh Seaside, Tinhgia	4	3	3	0	0	1	1	**13**	exploration
17	Dong Seaside, Tinhgia	5	6	5	2	1	2	4	**25**	involvement
	Nghe An province									
18	Dongthanh Seaside, Hoangmai Town	4	3	1	0	0	1	1	**10**	exploration
19	Quynhphuong Seaside, Hoangmai Town	5	6	5	2	1	2	4	**25**	involvement
20	Quynhlien Seaside, Quynhluu	4	3	2	0	0	1	1	**11**	exploration
21	Quynhbang Seaside. Quynhluu	5	4	4	0	1	2	1	**17**	exploration
22	Quynhluong Seaside, Quynhluu	4	3	2	0	0	1	1	**11**	exploration
23	Quynhminh Seaside, Quynhluu	4	3	2	0	0	1	1	**11**	exploration

24	Quynhnghia Seaside, Quynhluu	4	3	2	0	0	1	1	**11**	exploration
25	Tienthuy Seaside, Quynhluu	4	3	2	0	0	1	1	**11**	exploration
26	Lachvan Mangroves, Dienchau	3	1	1	0	0	1	1	**7**	exploration
27	Dienthanh Seaside, Dienchau	4	3	2	1	0	1	1	**12**	exploration
28	Cuahien Seaside, Dienchau	5	6	5	1	1	3	4	**25**	involvement
29	Bailu Seaside, Nghiloc	5	7	5	1	1	3	4	**26**	involvement
30	Cualo city	11	24	15	6	3	7	9	**75**	development
31	Cuahoi, Nghiloc	5	6	5	2	1	2	4	**25**	involvement
32	Hunghoa Mangroves (Vinh city)	3	1	1	0	0	1	1	**7**	exploration

Notes for table 2: 1: Tourist Resources (15p); 2: Visitors (30p); 3: Products and services (17p); 4: Management (8p); 5:Infrastructure (5p); 6: Local participation (10p); 7: Visitor Satisfaction (15p)

6. Recommendations

To develop tourism into a key economic sector in the economic structure of Thanh-Nghe coastal areas. Through this study, we suggest that the following solutions be implemented:

Planning solution. For planned tourist destinations, it is necessary to continue reviewing, supplementing and adjusting the planning in the direction of both developing tourism and protecting the natural, cultural and social environment. For unplanned destinations, general planning and detailed planning of tourist destinations should be carried out and published on most media.

Promote the diversification and uniquenes of tourism products and services associated with the natural, cultural and social characteristics of each coastal tourism site.

Infrastructure investment, especially transportation system, restaurants, hotels, etc. in the direction of harmonizing between popular tourism and high-grade tourism.

Enforcing promotion and linkages in tourism development. Especially there are solutions to overcome the seasonal difficulties in the sea and the island tourism.

Strengthening the management of tourism destinations which are at the stage of participation and development. Establishment of management boards in the destination is still in the exploration stage. Promoting short-term tourism training for people living in or near tourist destinations.

Enhancing the competitiveness of tourism destinations.

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EXPONENTIAL STABILITY FOR SINGULAR SYSTEMS WITH INTERVAL TIME-VARYING DELAYS

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Received: 12 February 2019 / Accepted: 11 June 2019 / Published: June 2019

Abstract: *This paper deals with the problem of exponential stability for singular systems with interval time-varying delays. By constructing a set of improved Lyapunov-Krasovskii functionals combined with Newton-Leibniz formula, a new delay-dependent condition is established in terms of linear matrix inequalities (LMIs) which guarantees that the system is regular, impulse-free and exponentially stability.*

Keywords: *Singular system; exponential stability; interval time-varying delay; linear matrix inequality.*

1. Introduction

Singular systems have many applications in reality such as electrical circuit network, power systems, aerospace engineering, network control, economic systems [6,19]. Therefore, these systems have been extensively studied over the past few decades and have yielded significant results. Using Lyapunov's approach to reach out to scientists who have established stable standards for singular systems with time-delay and singular systems with time-varying in the form of linear matrix inequalities [21,18]. However, there are few results concerned with the problem of exponential stability of singular systems with interval time-varying delays and most of the delay-dependent results in the literature tackled only the case of constant or slowly time-varying delays.

In this paper, a class of singular systems with interval time-varying delays is considered. New delay-range-dependent exponential stability condition is established in terms of LMIs ensuring the regularity, impulse free and exponential stability of the system. Employing the idea of perturbation approach, we decompose the system into slow and fast subsystems. Then, the exponential decay of slow variables is proved by constructing an improved LKF. Using this, we prove the fast variables fall within exponential decay with the same decay rate by some new estima- tions specifically developed in this paper. The main contribution of this paper is that we derive a new delay-range-dependent criterion for the exponential stability of singular systems with interval time-varying discrete delays.

Notations: The following notations will be used throughout this paper. $\mathbb{R}^+$ denotes the set of all nonnegative real numbers; $\mathbb{R}^n$ denotes the $n-$dimensional Euclidean space with

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the norm $\|.\|$ and scalar product $\langle x, y\rangle = x^{\mathrm{T}} y$ of two vectors $x, y;$ $\lambda_{\max}(A)\left(\lambda_{\min}(A), \text{resp.}\right)$ denotes the maximal (the minimal, resp.) number of the real part of eigenvalues of A; A^{T} denotes the transpose of the matrix A and I denotes the identity matrix; $Q \geq 0$ ($Q > 0$, resp.) means that Q is semi-positive definite (positive definite, resp.) i.e. $x^T Qx \geq 0$ for all $x \in \mathbb{R}^n$ (resp. $x^T Qx > 0$ for all $x \neq 0$); $A \geq B$ means $A - B \geq 0$; $C^1([-\tau, 0], \mathbb{R}^n)$ denotes the set of $\mathbb{R}^n$ - valued continuous functions on $[-\tau, 0]$ with the norm $\|\varphi\|_\tau = \sup_{-\tau \leq t \leq 0} \|\varphi(t)\|.$

2. Preliminaries

Consider the following singular system with time varying delays

$$\begin{cases} E\dot{x}(t) = & Ax(t) + Dx(t - h(t)), \quad t \geq 0, \\ x(t) = & \varphi(t), \quad t \in [-h_2, 0], \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the system state; $E, A, D \in \mathbb{R}^{n \times n}$ are real known system matrices with appropriate dimensions; matrix E may be singular with $\text{rank}(E) = r \leq n$. The time varying delays $h(t)$ is continuous functions satisfying $0 \leq h_1 \leq h(t) \leq h_2$ and $\dot{h}(t) \leq \mu < 1,$ where h_1 and h_2 are lower and upper bounds of the time varying delays $h(t).\phi(t) \in C\left([-h_2, 0], \mathbb{R}^n\right)$ is the compatible initial function specifying the intial state the system.

The following definitions for singular time delay system are adopted (e.g. see [18]).

Definition 1. [4], [21]

i) The pair (E, A) is said to be regular if the characteristic polynomial det (sE − A) is not identically.

ii) The pair (E, A) is said to be impulse free if deg(det(sE − A)) = rank(E).

Definition 2. [21]

i) System (1) is said to be regular and impulse-free if the pair (E, A) *regular and impulse.*

ii) $\alpha -$ *exponentially stable for* $\alpha > 0$ *if there exists* $N > 0$ *such that, for any compatible initial conditions* $\phi(t)$ *the solution* $x(t, \phi)$ *satisfies*

$$\|x(t, \varphi)\| \leq N \|\varphi\|_{h_2} e^{-\alpha t}, \quad \forall t \geq 0.$$

iii)Exponentially admissible if it is regular, impulse-free and $\alpha -$*exponentially stable.*

We introduce the following technical well-known propositions, which will be used in the proof of our results.

Proposition 1. [16] *(Matrix Cauchy inequality) For any* $M, N \in \mathbb{R}^{n \times n}$, $M = M^T > 0$ *and* $x, y \in \mathbb{R}^n$ *then* $2x^{\mathrm{T}} Ny \leq x^{\mathrm{T}} Mx + y^{\mathrm{T}} N^{\mathrm{T}} M^{-1} Ny.$

Proposition 2. [10] *For any symmetric positive definite matrix* M *, scalar* $\nu > 0$ *and vector function* $\omega : [0,\nu] \to \mathbb{R}^n$ *such that the integrals concerned are well defined, then*

$$\left(\int_0^{\nu} \omega(s)ds\right)^{\mathrm{T}} M \left(\int_0^{\nu} \omega(s)ds\right) \le \nu \int_0^{\nu} \omega^{\mathrm{T}}(s) M \omega(s) ds.$$

Proposition 3. [16] *(Schur complement Lemma) For given matrices* X, Y, Z *with appropriate dimensions satisfying* $X = X^{\mathrm{T}}, Y = Y^{\mathrm{T}} > 0.$ *Then* $\begin{bmatrix} X & Z^{\mathrm{T}} \\ Z & -Y \end{bmatrix} < 0$ *if and only if* $X + Z^{\mathrm{T}} Y^{-1} Z < 0.$

Lemma 1. *Let* $\tau > 0, \delta > 0, \gamma \in (0;1)$ *be given and* $\rho(t)$ *be a continuous function satisfying* $0 \le \rho(t) \le \gamma \bar{\rho}_{\tau}(t) + \delta, \forall t \ge 0$ *where* $\bar{\rho}_{\tau}(t) = \sup_{-\tau \le s \le 0} \rho(t+s).$ *Then* $\rho(t) \le \gamma \bar{\rho}_{\tau}(0) + \dfrac{\delta}{1-\gamma}, \quad \forall t \ge 0$.

Proof. Note that $\rho(0) \le \gamma \bar{\rho}_{\tau}(0) + \delta < \gamma \bar{\rho}_{\tau}(0) + \dfrac{\delta}{1-\gamma} =: \eta.$ We will prove $\rho(t) < \eta$ for all $t \ge 0$. Contrarily, assume that there exist $t_* > 0$ satisfying $\rho(t_*) = \eta, \rho(t) < \eta, \forall t \in [0, t_*),$ then $\rho(t_*) \le \eta,$ where $\rho(t) = \sup_{0 \le s \le t} \rho(s).$ From the fact that $\gamma\eta + \delta < \eta,$ we have $\rho(t_*) \le \gamma \max\{\bar{\rho}_{\tau}(0), \rho(t_*)\} + \delta \le \gamma \max\{\bar{\rho}_{\tau}(0), \eta\} + \delta < \eta$ which yields a contradiction. This shows that $\rho(t) \le \eta$ for all $t \le 0$.

By applying Lemma 1 for funtion $\rho(t) = e^{\lambda t} f(t)$ we obtain the following lemma.

Lemma 2. *Suppose that positive numbers* $\tau, \lambda, \delta_1, \delta_2, \delta_1 e^{\lambda\tau} < 1,$ *and continuous functions* $f(t)$ *satisfy* $0 \le f(t) \le \delta_1 \bar{f}_{\tau}(t) + \delta_2 e^{-\lambda t}, \quad \forall t \ge 0,$ *where* $\bar{f}_{\tau}(t) = \sup_{-\tau \le s \le 0} f(t+s).$ *Then* $f(t) \le \left[\delta_1 e^{\lambda\tau} \bar{f}_{\tau}(0) + \dfrac{\delta_2}{1-\delta_1 e^{\lambda\tau}}\right] e^{-\lambda t}, \quad \forall t \ge 0.$

3. A main result

For given $\alpha > 0$. We denote:

$$\Pi_{11} = A^{\mathrm{T}} P^{\mathrm{T}} + PA + Q + Q_1 + Q_2 + 2\alpha PE + X_1 E + E^{\mathrm{T}} X_1^{\mathrm{T}}, \Pi_{12} = PD - X_1 E + E^{\mathrm{T}} X_2^{\mathrm{T}} + Y_1 E - Z_1 E,$$

$$\Pi_{18} = A^{\mathrm{T}}[h_2 W_1 + (h_2 - h_1) W_2], \Pi_{22} = -(1-\mu) e^{-2\alpha h_2} Q + Y_2 E + E^{\mathrm{T}} Y_2^{\mathrm{T}} - X_2 E - E^{\mathrm{T}} X_2^{\mathrm{T}} - Z_2 E - E^{\mathrm{T}} Z_2^{\mathrm{T}},$$

$$\Pi_{28} = D^{\mathrm{T}}[h_2 W_1 + (h_2 - h_1) W_2], \Pi_{33} = -e^{-2\alpha h_1} Q_1, \Pi_{44} = -e^{-2\alpha h_2} Q_2, \Pi_{66} = -\gamma_{22}(W_1 + W_2)$$

$$\Pi_{88} = -[h_2 W_1 + (h_2 - h_1) W_2], \gamma_{21} = \frac{e^{2\alpha h_2} - 1}{2\alpha}, \gamma_{22} = \frac{e^{2\alpha h_2} - e^{2\alpha h_1}}{2\alpha}.$$

Theorem 1. *Given* $\alpha > 0$. *System (1) is* $\alpha -$*exponentially admissible if there exist symmetric positive definite matrices* Q, Q_i, W_i $i = 1,2,$ *and matrices* $P, X_i, Y_i, Z_i,$ $i = 1,2,$ *satisfying the following LMIs:*

$$PE = E^{\mathrm{T}} P^{\mathrm{T}} \geq 0, \tag{2}$$

$$\begin{bmatrix} \Pi_{11} & \Pi_{12} & Z_1 E & -Y_1 E & \gamma_{21} X_1 & \gamma_{22} Y_1 & \gamma_{22} Z_1 & \Pi_{18} \\ * & \Pi_{22} & Z_2 E & -Y_2 E & \gamma_{21} X_2 & \gamma_{22} Y_2 & \gamma_{22} Z_2 & \Phi_{28} \\ * & * & \Pi_{33} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Pi_{44} & 0 & 0 & 0 & 0 \\ * & * & * & * & -\gamma_{21} W_1 & 0 & 0 & 0 \\ * & * & * & * & * & \Pi_{66} & 0 & 0 \\ * & * & * & * & * & * & -\gamma_{22} W_2 & 0 \\ * & * & * & * & * & * & * & \Pi_{88} \end{bmatrix} < 0, \tag{3}$$

$$(1-\mu)Q - Q_2 < 0. \tag{4}$$

Proof. Step 1: We prove the regularity and impulse-free of system (1).

Since $\mathrm{rank}(E) = r \leq n$, these exists two nonsingular matrices M, N such that $\bar{E} = MEN = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$. We denote

$$\bar{A} = MAN = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \bar{P} = N^{\mathrm{T}} P M^{-1} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}, \bar{Q} = N^{\mathrm{T}} Q N = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}^{\mathrm{T}} & Q_{22} \end{bmatrix},$$

$$\bar{Q}_i = N^{\mathrm{T}} Q_i N = \begin{bmatrix} Q_{i11} & Q_{i12} \\ Q_{i12}^{\mathrm{T}} & Q_{i22} \end{bmatrix}, \bar{X}_i = N^{\mathrm{T}} X_i M^{-1} = \begin{bmatrix} X_{i11} & X_{i12} \\ X_{i21} & X_{i22} \end{bmatrix}, \bar{Y}_i = N^{\mathrm{T}} Y_i M^{-1} = \begin{bmatrix} Y_{i11} & Y_{i12} \\ Y_{i21} & Y_{i22} \end{bmatrix},$$

$$\bar{Z}_i = N^{\mathrm{T}} Z_i M^{-1} = \begin{bmatrix} Z_{i11} & Z_{i12} \\ Z_{i21} & Z_{i22} \end{bmatrix}, \quad \bar{W}_i = M^{-\mathrm{T}} W_i M^{-1} = \begin{bmatrix} W_{i11} & W_{i12} \\ W_{i12}^{\mathrm{T}} & W_{i22} \end{bmatrix}, i = 1,2.$$

From (2) we have

$$\bar{P}\bar{E} = \bar{E}^{\mathrm{T}} \bar{P}^{\mathrm{T}} \geq 0. \tag{5}$$

Using the expression of $\bar{E}$ and $\bar{P}$, we obtain $P_{21} = 0, P_{11} \geq 0$. From (3) using the Proposition 3, we have

$$\begin{bmatrix} \Pi_{11} & \Pi_{12} & Z_1 E & -Y_1 E \\ * & \Pi_{22} & Z_2 E & -Y_2 E \\ * & * & \Pi_{33} & 0 \\ * & * & * & \Pi_{44} \end{bmatrix} < 0, \tag{6}$$

Pre-multiplying $\begin{bmatrix} I & I & I & I \end{bmatrix}$ and post-multiplying $\begin{bmatrix} I & I & I & I \end{bmatrix}^{\mathrm{T}}$ on both sides of (6), we obtain

$$(A+D)^{\mathrm{T}} P^{\mathrm{T}} + P(A+D) + 2\alpha PE + (1-e^{-2\alpha h_1})Q_1 + (1-e^{-2\alpha h_2})[(1-\mu)Q + Q_2] < 0 \quad (7)$$

and hence $(A+D)^{\mathrm{T}} P^{\mathrm{T}} + P(A+D) < 0$. By Lemma 3, which proves that P is nonsingular, and hence $\bar{P}$ is nonsingular, then $P_{11} > 0$. On the other hand, from $\begin{bmatrix} \Pi_{11} & \Pi_{12} \\ * & \Pi_{22} \end{bmatrix} < 0$ pre-multiplying $\mathrm{diag}\{N^{\mathrm{T}}, N^{\mathrm{T}}\}$ and post-multiplying $\mathrm{diag}\{N, N\}$ we obtain

$$\begin{bmatrix} \bar{\Pi}11 & \bar{\Pi}12 \\ * & \bar{\Pi}22 \end{bmatrix} < 0 \quad (8)$$

where

$$\bar{\Pi}_{11} = \bar{A}^{\mathrm{T}}\bar{P}^{\mathrm{T}} + \bar{P}\bar{A} + \bar{Q} + \bar{Q}_1 + \bar{Q}_2 + 2\alpha\bar{P}\bar{E} + \bar{X}_1\bar{E} + \bar{E}^{\mathrm{T}}\bar{X}_1^{\mathrm{T}}, \bar{\Pi}_{12} = \bar{P}\bar{D} - \bar{X}_1\bar{E} + \bar{E}^{\mathrm{T}}\bar{X}_2^{\mathrm{T}} + \bar{Y}_1\bar{E} - \bar{Z}_1\bar{E},$$

$$\bar{\Pi}_{22} = -(1-\mu)e^{-2\alpha h_2}\bar{Q} + \bar{Y}_2\bar{E} + \bar{E}^{\mathrm{T}}\bar{Y}_2^{\mathrm{T}} - \bar{X}_2\bar{E} - \bar{E}^{\mathrm{T}}\bar{X}_2^{\mathrm{T}} - \bar{Z}_2\bar{E} - \bar{E}^{\mathrm{T}}\bar{Z}_2^{\mathrm{T}}$$

Applying Lemma 4, from (4) we obtain

$$\begin{bmatrix} P_{22}A_{22} + A_{22}^{\mathrm{T}}P_{22}^{\mathrm{T}} + Q_{22} + Q_{122} + Q_{222} & P_{22}D_{22} \\ * & -(1-\mu)e^{-2\alpha h_2}Q_{22} \end{bmatrix} < 0, \quad (9)$$

Which gives $P_{22}A_{22} + A_{22}^{\mathrm{T}}P_{22}^{\mathrm{T}} < 0,$ and hence P_{22} and A_{22} are nonsingular matrices. Implies system (1) is regular and impulse free.

Next, we can choose two nonsingular matrices M, N such that $\bar{E} = MEN = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$ and $\bar{A} = MAN = \begin{bmatrix} A_{11} & 0 \\ 0 & I_{n-r} \end{bmatrix}$.

Step 2: Decompose the system and exponential estimate for slow variables. Under variable transformation

$$y(t) = N^{-1}x(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix}, \quad (10)$$

where $y_1(t) \in \mathbb{R}^r, y_2(t) \in \mathbb{R}^{n-r}$. The corresponding transformed system (2.3) is given by

$$\begin{cases} \bar{E}\dot{y}(t) & = \bar{A}y(t) + \bar{D}y(t-h(t)), \quad t \ge 0, \\ y(t) & = N^{-1}\varphi(t) := \psi(t), \quad t \in [-h_2, 0], \end{cases} \quad (11)$$

In other word, under the transformation $y(t) = N^{-1}x(t)$, system (1) is decomposed into the following slow and fast subsystems

$$\dot{y}_1(t) = A_{11}y_1(t) + D_{11}y_1(t-h(t)) + D_{12}y_2(t-h(t)), \quad (12)$$

$$0 = y_2(t) + D_{21}y_1(t-h(t)) + D_{22}y_2(t-h(t)) \tag{13}$$

System (12) and (13) are referred to as slow and fast subsystems and $y_1(t) \in \mathbb{R}^r, y_2 \in \mathbb{R}^{n-r}$ are called slow and fast variables, respectivily. We now prove the exponential stability of slow subsystem (8). For this, we construct the following LKF

$$V(y_t) = V_1 + V_2 + V_3 + V_4 + V_5 + V_6 \tag{14}$$

where

$$V_1 = y^{\mathrm{T}}(t)\bar{P}\bar{E}y(t), \qquad V_4 = \int_{t-h(t)}^{t} e^{2\alpha(s-t)} y^{\mathrm{T}}(s)\bar{Q}y(s)ds,$$

$$V_2 = \int_{t-h_1}^{t} e^{2\alpha(s-t)} y^{\mathrm{T}}(s)\bar{Q}_1 y(s)ds, \qquad V_5 = \int_{-h_2}^{0}\int_{t+s}^{t} e^{2\alpha(u-t)} \dot{y}^{\mathrm{T}}(u)\bar{E}^{\mathrm{T}}\bar{W}_1\bar{E}\dot{y}(u)duds,$$

$$V_3 = \int_{t-h_2}^{t} e^{2\alpha(s-t)} y^{\mathrm{T}}(s)\bar{Q}_2 y(s)ds, \qquad V_6 = \int_{-h_2}^{-h_1}\int_{t+s}^{t} e^{2\alpha(u-t)} \dot{y}^{\mathrm{T}}(u)\bar{E}^{\mathrm{T}}\bar{W}_2\bar{E}\dot{y}(u)duds.$$

It is easy to see that

$$\lambda_1 \left\| y_1(t) \right\|^2 \le V(t, y_t) \le \lambda_2 \left\| y_t \right\|^2, \tag{15}$$

where y_t denotes the segment $\{y(t+s) : s \in [-h_2; 0]\}, \lambda_1 = \lambda_{\min}(P_{11})$ and

$$\lambda_2 = \lambda_{\max}(P_{11}) + h_1\lambda_{\max}(\bar{Q}_1) + h_2\lambda_{\max}(\bar{Q}_2) + h_2\lambda_{\max}(\bar{Q}) + \left[h_2^2\lambda_{\max}(\bar{W}_1) + (h_2^2 - h_1^2)\lambda_{\max}(\bar{W}_2) \right].$$

Taking derivative of V_1 in t along the trajectory of the system, we have

$$\begin{aligned}\dot{V}_1 &= 2y^{\mathrm{T}}(t)\bar{P}\bar{E}\dot{y}(t) = 2y^{\mathrm{T}}(t)\bar{P} \times \left[\bar{A}y(t) + \bar{D}y(t-h(t)) \right] \\ &= y^{\mathrm{T}}(t)\left[\bar{P}\bar{A} + \bar{A}^{\mathrm{T}}\bar{P}^{\mathrm{T}} + 2\alpha\bar{P}\bar{E} \right] y(t) + 2y^{\mathrm{T}}(t)\bar{P}\bar{D}y(t-h(t)) - 2\alpha V_1.\end{aligned} \tag{16}$$

The time-derivative of $V_k, k = 2, 3, \ldots, 8,$ are computed and estimated as follows

$$\begin{aligned}\dot{V}_2 &= y^{\mathrm{T}}(t)\bar{Q}_1 y(t) - e^{-2\alpha h_1} y^{\mathrm{T}}(t-h_1)\bar{Q}_1 y(t-h_1) - 2\alpha V_2; \\ \dot{V}_3 &= y^{\mathrm{T}}(t)\bar{Q}_2 y(t) - e^{-2\alpha h_2} y^{\mathrm{T}}(t-h_2)\bar{Q}_2 y(t-h_2) - 2\alpha V_3; \\ \dot{V}_4 &= y^{\mathrm{T}}(t)\bar{Q}y(t) - (1-\dot{h}(t))e^{-2\alpha h(t)} y^{\mathrm{T}}(t-h(t))\bar{Q}y(t-h(t)) - 2\alpha V_4 \\ &\le y^{\mathrm{T}}(t)\bar{Q}y(t) - (1-\mu)e^{-2\alpha h_2} y^{\mathrm{T}}(t-h(t))\bar{Q}y(t-h(t)) - 2\alpha V_4;\end{aligned} \tag{17}$$

$$\begin{aligned}\dot{V}_5 &= h_2\dot{y}^{\mathrm{T}}(t)\bar{E}^{\mathrm{T}}\bar{W}_1\bar{E}\dot{y}(t) - \int_{t-h_2}^{t} e^{2\alpha(s-t)} \dot{y}^{\mathrm{T}}(s)\bar{E}^{\mathrm{T}}\bar{W}_1\bar{E}\dot{y}(s)ds - 2\alpha V_7, \\ \dot{V}_6 &= (h_2 - h_1)\dot{y}^{\mathrm{T}}(t)\bar{E}^{\mathrm{T}}\bar{W}_2\bar{E}\dot{y}(t) - \int_{t-h_2}^{t-h_1} e^{2\alpha(s-t)} \dot{y}^{\mathrm{T}}(s)\bar{E}^{\mathrm{T}}\bar{W}_2\bar{E}\dot{y}(s)ds - 2\alpha V_8.\end{aligned} \tag{18}$$

Using the following identities

$$2\left[y^{\mathrm{T}}(t)\bar{X}_1+y^{\mathrm{T}}(t-h(t))\bar{X}_2\right]\left[\bar{E}y(t)-\bar{E}y(t-h(t))-\int_{t-h(t)}^{t}\bar{E}\dot{y}(s)ds\right]=0, \tag{19}$$

$$2\left[y^{\mathrm{T}}(t)\bar{Y}_1+y^{\mathrm{T}}(t-h(t))\bar{Y}_2\right]\left[\bar{E}y(t-h(t))-\bar{E}y(t-h_2)-\int_{t-h_2}^{t-h(t)}\bar{E}\dot{y}(s)ds\right]=0,$$

$$2\left[y^{\mathrm{T}}(t)\bar{Z}_1+y^{\mathrm{T}}(t-h(t))\bar{Z}_2\right]\left[\bar{E}y(t-h_1)-\bar{E}y(t-h(t))-\int_{t-h(t)}^{t-h}\bar{E}\dot{y}(s)ds\right]=0.$$

From (16) to (19), we have

$$\dot{V}(t,y_t)+2\alpha V(t,y_t)\le \eta^T(t)\bar{\Pi}\eta(t)+\eta^T(t)\Omega^{\mathrm{T}}\bar{U}\Omega\eta(t)+\frac{e^{2\alpha h_2}-e^{2\alpha h_1}}{2\alpha}\eta^{\mathrm{T}}(t)\bar{Y}(\bar{W}_1+\bar{W}_2)^{-1}\bar{Y}^{\mathrm{T}}\eta(t) \tag{20}$$

$$+\frac{e^{2\alpha h_2}-e^{2\alpha h_1}}{2\alpha}\eta^{\mathrm{T}}(t)\bar{Z}\bar{W}_2^{-1}\bar{Z}^{\mathrm{T}}\eta(t)+\frac{e^{2\alpha h_2}-1}{2\alpha}\eta^{\mathrm{T}}(t)\bar{X}\bar{W}_1^{-1}\bar{X}^{T}\eta(t)$$

$$\le \eta^{\mathrm{T}}(t)[\bar{\Pi}+\gamma_{21}\bar{X}\bar{W}_1^{-1}\bar{X}^{\mathrm{T}}+\gamma_{22}\bar{Z}\bar{W}_2^{-1}\bar{Z}^{\mathrm{T}}+\gamma_{22}\bar{Y}(\bar{W}_1+\bar{W}_2)^{-1}\bar{Y}^{\mathrm{T}}+\Omega^{\mathrm{T}}\bar{U}\Omega]\eta(t),$$

where

$$\eta^{\mathrm{T}}(t)=\left[y^{\mathrm{T}}(t)\quad y^{\mathrm{T}}(t-h(t))\quad y^{\mathrm{T}}(t-h_1)\quad y^{\mathrm{T}}(t-h_2)\right],\bar{X}=\left[\bar{X}_1^{\mathrm{T}}\quad \bar{X}_2^{\mathrm{T}}\quad 0\quad 0\right]^{\mathrm{T}},\quad \bar{Y}=\left[\bar{Y}_1^{\mathrm{T}}\quad \bar{Y}_2^{\mathrm{T}}\quad 0\quad 0\right]^{\mathrm{T}},$$

$$\bar{Z}=\left[\bar{Z}_1^{\mathrm{T}}\quad \bar{Z}_2^{\mathrm{T}}\quad 0\quad 0\right]^{\mathrm{T}},\quad \bar{U}=h_2\bar{W}_1+(h_2-h_1)\bar{W}_2,\Omega=\left[\bar{A}\quad \bar{D}\quad 0\quad 0\right],$$

and $\bar{\Pi}=\begin{bmatrix}\bar{\Pi}_{11} & \bar{\Pi}_{12} & \bar{Z}_1\bar{E} & -\bar{Y}_1\bar{E}\\ * & \bar{\Pi}_{22} & \bar{Z}_2\bar{E} & -\bar{Y}_2\bar{E}\\ * & * & -e^{2\alpha h_1}\bar{Q}_1 & 0\\ * & * & * & -e^{2\alpha h_2}\bar{Q}_2\end{bmatrix}$

On the other hand, by pre-multiplying $\mathrm{diag}\{N^{\mathrm{T}},N^{\mathrm{T}},N^{\mathrm{T}},N^{\mathrm{T}},I,I,I,I\}$ and post-multiplying $\mathrm{diag}\{N,N,N,N,I,I,I,I\}$ on both sides of (3) , we obtain

$$\begin{bmatrix}\bar{\Pi}_{11} & \bar{\Pi}_{12} & \bar{Z}_1\bar{E} & -\bar{Y}_1\bar{E} & \gamma_{21}N^{\mathrm{T}}X_1 & \gamma_{22}N^{\mathrm{T}}Y_1 & \gamma_{22}N^{\mathrm{T}}Z_1 & N^{\mathrm{T}}\Pi_{18}\\ * & \bar{\Pi}_{22} & \bar{Z}_2\bar{E} & -\bar{Y}_2\bar{E} & \gamma_{21}N^{\mathrm{T}}X_2 & \gamma_{22}N^{\mathrm{T}}Y_2 & \gamma_{22}N^{\mathrm{T}}Z_2 & N^{\mathrm{T}}\Pi_{28}\\ * & * & -e^{2\alpha h_1}\bar{Q}_1 & 0 & 0 & 0 & 0 & 0\\ * & * & * & -e^{2\alpha h_2}\bar{Q}_2 & 0 & 0 & 0 & 0\\ * & * & * & * & -\gamma_{21}W_1 & 0 & 0 & 0\\ * & * & * & * & * & \Pi_{66} & 0 & 0\\ * & * & * & * & * & * & -\gamma_{22}W_2 & 0\\ * & * & * & * & * & * & * & \Pi_{88}\end{bmatrix}<0. \tag{21}$$

By using Proposition 3 for (21), it can be shown that

$$\bar{\Phi}+\gamma_{21}\bar{X}\bar{W}_1^{-1}\bar{X}^{\mathrm{T}}+\gamma_{22}\bar{Z}\bar{W}_2^{-1}\bar{Z}^{\mathrm{T}}+\gamma_{22}\bar{Y}(\bar{W}_1+\bar{W}_2)^{-1}\bar{Y}^{\mathrm{T}}+\Omega^{\mathrm{T}}\bar{U}\Omega<0. \quad (22)$$

From (20) and (22), we have

$$\dot{V}(t,y_t)+2\alpha V(t,y_t)\le 0, t\ge 0$$

and hence

$$V(y_t)\le V(\psi)e^{-2\alpha t}\le \lambda_2\|\psi\|_{h_2}^2 e^{-2\alpha t}, \ge 0.$$

Taking (15) into account, we obtain

$$\|y_1(t)\|\le\sqrt{\frac{\lambda_2}{\lambda_1}}\|\psi\|_{h_2}e^{-\alpha t}=:\upsilon_1\|\psi\|_{h_2}e^{-\alpha t},\quad t\ge 0. \quad (23)$$

This proves that the slow variable, that is, the first $r-$dimensional component $y_1(t)$ of the state vector $y(t)$ is $\alpha-$ exponentially stable.

Step 3: The exponential decay of fast variables. In this step, we will prove the fast variables are fallen into exponential decay with the same decay rate α. Let us denote $p(t)=A_{h21}y_1(t-h(t))$ by pre-multiplying the second equation of (8) with $2y_2^{\mathrm{T}}(t)P_{22}$, we obtain

$$0=2y_2^{\mathrm{T}}(t)P_{22}y_2(t)+2y_2^{\mathrm{T}}(t)P_{22}D_{22}y_2(t-h(t))+2y_2^{\mathrm{T}}(t)P_{22}p(t). \quad (24)$$

Consider the following function

$$J(t):=(1-\mu)y_2^{\mathrm{T}}(t)Q_{22}y_2(t)-(1-\mu)e^{-2\alpha h_2}y_2^{\mathrm{T}}(t-h(t))Q_{22}y_2(t-h(t)) \quad (25)$$

Then, from (24) we have,

$$J(t)=y_2^{\mathrm{T}}(t)[P_{22}+P_{22}^{\mathrm{T}}+(1-\mu)Q_{22}]y_2(t)+2y_2^{\mathrm{T}}(t)P_{22}p(t)$$
$$-(1-\mu)e^{-2\alpha h_2}y_2^{\mathrm{T}}(t-h(t))Q_{22}y_2(t-h(t))+2y_2^{\mathrm{T}}(t)P_{22}D_{22}y_2(t-h(t)).$$

Applying Proposition 1 to this yields that,

$$J(t)\le y_2^{\mathrm{T}}(t)[P_{22}+P_{22}^{\mathrm{T}}+(1-\mu)Q_{22}]y_2(t)+y_2^{\mathrm{T}}(t)Q_{122}y_2(t)+p^{\mathrm{T}}(t)P_{22}^{\mathrm{T}}Q_{122}^{-1}P_{22}p(t) \quad (26)$$
$$+2y_2^{\mathrm{T}}(t)P_{22}D_{22}y_2(t-h(t))-(1-\mu)e^{-2\alpha h_2}y_2^{\mathrm{T}}(t-h(t))Q_{22}y_2(t-h(t))$$
$$\le\begin{bmatrix}y_2(t)\\ y_2(t-h(t))\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}J_{11} & P_{22}D_{22}\\ & -(1-\mu)e^{-2\alpha h_2}Q_{22}\end{bmatrix}\begin{bmatrix}y_2(t)\\ y_2(t-h(t))\end{bmatrix}+p^{\mathrm{T}}(t)P_{22}^{\mathrm{T}}Q_{122}^{-1}P_{22}p(t)$$

where $J_{11}=P_{22}+P_{22}^{\mathrm{T}}+(1-\mu)Q_{22}+Q_{122}$. On the other hand, from (4) it follows

$$\begin{bmatrix}J_{11}+Q_{222} & P_{22}D_{22}\\ * & -(1-\mu)e^{-2\alpha h_2}Q_{22}\end{bmatrix}<0.$$

Hence, from (26) we have

$$J(t)\le -y_2^{\mathrm{T}}(t)Q_{222}y_2(t)+p^{\mathrm{T}}(t)P_{22}^{\mathrm{T}}Q_{122}^{-1}P_{22}p(t) \quad (27)$$

It follows from (25), (27) and applying propositon 2, we have

$$y_2^{\mathrm{T}}(t)[(1-\mu)Q_{22}+Q_{222}]y_2(t)\le(1-\mu)y_2^{\mathrm{T}}(t-h(t))e^{-2\alpha h_2}Q_{22}y_2(t-h(t))+p^{\mathrm{T}}(t)P_{22}^{\mathrm{T}}Q_{122}^{-1}P_{22}p(t) \quad (28)$$

By pre - and post - multiplying with N^{T}, N, it follows from (4) $(1-\mu)Q_{22} < Q_{222}$. Therefore

$$2(1-\mu)y_2^{\mathrm{T}}(t)Q_{22}y_2(t) \le (1-\mu)e^{-2\alpha h_2} \sup_{-h_2\le s\le 0} y^{\mathrm{T}}(t+s)Q_{22}y(t+s) + p^{\mathrm{T}}(t)P_{22}^{\mathrm{T}}Q_{122}^{-1}P_{22}p(t) \qquad (29)$$

Observe that, for all $t \ge 0$, if $t-h(t)\ge 0$ then

$$\|y_1(t-h(t))\|^2 \le \frac{\lambda_2}{\lambda_1}\|\psi\|_{h_2}^2 e^{-2\alpha(t-h(t))} \le \frac{\lambda_2}{\lambda_1}\|\psi\|_{h_2}^2 e^{2\alpha h_2}e^{-2\alpha t}.$$

Otherwise, $\|y_1(t-h(t))\|^2 \le \|\psi\|_{h_2}^2 \le \|\psi\|_{h_2}^2 e^{-2\alpha(t-h(t))} \le \|\psi\|_{h_2}^2 e^{2\alpha h_2}e^{-2\alpha t}$ Therefore

$$\|y_1(t-h(t))\| \le \upsilon_1 e^{\alpha h_2}\|\psi\|_{h_2} e^{-\alpha t}, \quad t\ge 0.$$

From (29), we obtain

$$2(1-\mu)y_2^{\mathrm{T}}(t)Q_{22}y_2(t) \le (1-\mu)e^{-2\alpha h_2} \sup_{-h_2\le s\le 0} y^{\mathrm{T}}(t+s)Q_{22}y(t+s) + \frac{\lambda_{max}(P_{22}^{\mathrm{T}}P_{22})}{\lambda_{min}(Q_{122})}\left[\|D_{21}\|e^{\alpha h_2}e^{-\alpha t}\right]^2 \upsilon_1^2\|\psi\|_{h_2}^2$$

Let us denote $f(t) = y_2^{\mathrm{T}}(t)Q_{22}y_2(t), \quad t\ge -h_2$, and $\bar{f}_{h_2}(t) = \sup_{-h_2\le s\le 0} f(t+s)$, then

$$f(t) \le \delta_1 \bar{f}_{h_2}(t) + \delta_2 e^{-2\alpha t}, \quad t \ge 0 \qquad (30)$$

where

$$\delta_1 = \frac{e^{-2\alpha h_2}}{2}, \delta_2 = \frac{\lambda_{max}\left(P_{22}^{\mathrm{T}}P_{22}\right)\|D_{21}\|^2 e^{2\alpha h_2}\upsilon_1^2\|\psi\|_{h_2}^2}{2\lambda_{min}\left(Q_{122}\right)(1-\mu).}.$$

Note that $\tau_{(\alpha,h)} = (1-\mu)$, hence

$$\delta_1 e^{2\alpha h_2} = \frac{e^{-2\alpha h_2}e^{2\alpha h_2}}{2} = \frac{1}{2} < 1.$$

By Lemma 2, it follows from (30) that

$$f(t) \le \left[\delta_1 e^{2\alpha h_2}\bar{f}_{h_2}(0) + \frac{\delta_2}{1-\delta_1 e^{2\alpha h_2}}\right]e^{-2\alpha t}, \quad t\ge 0.$$

Furthermore, it can be verified that

$$\bar{f}_{h_2}(0) \le \lambda_{max}(Q_{22})\|\psi\|_{h_2}^2, \quad \frac{\delta_2}{1-\delta_1 e^{2\alpha h_2}} = \frac{\lambda_{max}\left(P_{22}^{\mathrm{T}}P_{22}\right)\|D_{21}\|^2 e^{2\alpha h_2}\upsilon_1^2\|\psi\|_{h_2}^2}{\lambda_{min}\left(Q_{122}\right)(1-\mu)}.$$

And thus

$$\lambda_{min}\left(Q_{22}\right)\|y_2(t)\|^2 \le \left[\frac{\lambda_{max}(Q_{22})}{2} + \frac{\lambda_{max}\left(P_{22}^{\mathrm{T}}P_{22}\right)\left(D_{21}\right]^2 e^{2\alpha h_2}\upsilon_1^2}{\lambda_{min}\left(Q_{122}\right)(1-\mu)}\right]\|\psi\|_{h_2}^2 e^{-2\alpha t}, \quad t\ge 0.$$

Consequently

$$\|y_2(t)\| \le \upsilon_2\|\psi\|_{h_2} e^{-\alpha t}, \quad t\ge 0$$

where

$$\upsilon_2 = \left[\frac{\lambda_{max}\left(Q_{22}\right)}{2} + \frac{\lambda_{max}\left(P_{22}^{\mathrm{T}}P_{22}\right)\left\|D_{21}\right\|^2 e^{2\alpha h_2}\upsilon_1^2}{\lambda_{min}\left(Q_{122}\right)(1-\mu)} \right]^{\frac{1}{2}}.$$

This shows that the fast variable $y_2(t)$ is fallen into exponential decay with decay rate α. Note that $x(t) = Ny(t)$ we readily obtain

$$\|x(t,\varphi)\| \le \upsilon \|\varphi\|_{h_2} e^{\alpha t}, \quad t \ge 0$$

where $\upsilon = \|N\|\|N\|^{-1}\sqrt{\upsilon_1^2 + \upsilon_2^2}$. This completes the proof.

Remark: The result of this paper has resolved exponential stability for singular systems with interval time-varying delays while in the [21] tackled only the case of constant. Also, there were some results have given the exponential stability standards but it only applies to nonsingular systems (see. [5, 11, 20]). Therefore, the result in Theorem 1 is more advanced than the other paper' results.

4. Conslusion

In this paper, we have studied the exponential stability of singular systems with interval time-varying delays. On the basis of the new lemma and constructing a set of new Lyapunov-Krasovskii functionals, sufficient conditions for the exponential stability is established in terms of LMIs which guarantee that the system is regular, impulse-free and exponentially stability.

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AN OVERVIEW OF HISTORY, CULTURE AND SOCIETY OF HMONG PEOPLE IN THANH HOA

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Received: 12 February 2019 / Accepted: 11 June 2019 / Published: June 2019

Abstract: *Hmong people belong to the Hmong - Yao language branch, the South Asian language family, is one of the ethnic minority groups which has a large number of population among ethnic groups in Vietnam. Hmong people have the slow migration and settlement process compared to other ethnic groups in our country. They are located mainly in some highland provinces: Ha Giang, Tuyen Quang, Dien Bien, Lai Chau, Cao Bang, Lang Son, Lao Cai, Son La, etc. and in the western mountainous region of Thanh Hoa and Nghe An provinces. Hmong people in Thanh Hoa reside mainly in three districts: Muong Lat, Quan Son and Quan Hoa. Most of them come from other places in the northern mountainous areas, migrating through two direct ways. Essentially, Hmong people in Thanh Hoa have a high sense of ethnicity and also maintain many traditional cultural features which contribute to enlivening the colorful picture of ethnic culture of Xu Thanh.*

Keywords: *Hmong people, Thanh Hoa, history, culture and society.*

1. Introduction

Among the ethnic minority community of Vietnam, the Hmong people belong to the Hmong - Yao language branch, of the South Asian language family; is one of the largest ethnic minority groups in our country (recently Hmong population around 1,068,189) [2].

The name Hmong has a long historic origin. The name HMong following the Chinese-Vietnamese pronunciation is Miao (Miêu). According to some researchers, this name is used to call the people who early knew the cultivation of rice in area of Banh Loi and Dong Dinh lakes, which has now become an official name. The majority of Hmong people in our country call themselves as *Mống* (Hmoob), other Hmong groups call themselves as *Ná Miảo*. Mống is also the name of Hmong people in Laos, Thailand and in some parts of China. The name-Mống means the name of ethnic communities as well as the name of Family. Based on a number of ethnographic characteristics, it is possible to divide Hmong people into some groups: Hmong Trang (Mongz Dou), Hmong Hoa (Mongz Lenhs), Hmong Xanh (Mongz Dua), Hmong Den (Mongz Duz).

Hmong people are considered as the descendants of one of the ancient indigenous inhabitants living in south China. From southwest China, Hmong people migrated to Vietnam in different times. The reasons of the migration is the desire to escape the oppression of feudal landlords, the hope to escape the slaughter after the protests and having a better place to live. The

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first migrations of the Hmong people began over 300 years ago to the border areas of Ha Giang and Tuyen Quang province. Then, the migration flow lasted until the day China was completely liberated in 1950, in which there were two large migrations about 200 years and 100 years ago. Most of our Hmong people still remember that they came from Quy Chau to Vietnam [5].

Currently, Hmong people in Vietnam live mainly in some provinces such as Ha Giang, Tuyen Quang, Dien Bien, Lai Chau, Cao Bang, Lang Son, Lao Cai, Son La ... and the western mountainous regions of Thanh Hoa and Nghe An provinces.

The habitat areas of Hmong people are often on the mountainside with an average altitude from 800m to 1500-1700m, rugged terrain, cliffs erect on the valleys of abyss areas. At an altitude of 800m - 1700m, the climate lies in the sub-tropical climate, the annual average temperature is from 15- 20 celsius. Annual rainfall is in the range of 2000mm- 3000mm, however, in some places where are sheltered from the wind, rainfall is only about 700mm. Characteristics of rainfall where Hmong people live in are a rainy season and a dry season. In the dry season, monthly rainfall is only tens of millimeters or less. Sometimes it does not rain, so this area often lacks of water. With the diligence and creativeness, Hmong people have turned many highlands of northern Vietnam into their beloved homeland. For Hmong people in Sa Pa - Lao Cai, they possess an extremely rich and diverse terraced field system. This is a wonderful and unique creative work which is very valuable - as the unique cultural heritage, preserved through many generations of Hmong people. Terraced fields are a treasure of indigenous knowledge of ethnic groups that need to be preserved and promoted. It can be seen that the imprint of the Hmong people associated with the life of settlement and sustainable development for many generations in this important frontier. In order to affirm the land of Lao Cai or Ha Giang are their beloved second hometown. Hmong people often say:

The fish live in the water (Cá ở dưới nước)
The birds fly in the sky (Chim bay ở trên trời)
We live in the highland (Chúng ta sống ở vùng cao)
And the birds have their nests (Và con chim có tổ)
Hmong people ourselves also have the root (Người Mông ta cũng có quê)
Our root is in Mong Vac (Quê ta là Mông Vạc) [5]

People in Dong Van plateau also believe that on the top of Mong Vac mountain, there is a God's well and there is a couple of birds specialized in picking up leaves to make water in God's well clean. People want to drink the water from the well to get away from sickness and when they die, they will be returned to their ancestors. Therefore, in the past, many old people wished to visit Mong Vac, "where the fatherland was held" and "to drink miraculous water from the well". Besides the superstitious factor, these things have a certain meaning to the formation of the love of the homeland and also arising the patriotism of Vietnam of the Hmong people. That patriotism is increasingly strengthened in the process of standing side by side with ethnic people fighting to protect our country and build a new life.

2. History and ethnic process of Hmong people in Thanh Hoa

About the origin of Hmong people in Thanh Hoa, in a research towards Thanh Hoa province (Le Province de Thanh Hoa) written by H. Le Breton (1918) describles that some people met a number of Hmong people and Yao people in mountainous area of Thanh Hoa.

Those were the new residents from China traveling to Indochina most recently around the middle of the last century (19th century), apparently after the Chinese slaughter (under the Qing Dynasty) at that time in Guizhou region. Also, according to H. Le Breton, Hmong people are very good at tilling a field (on the mountain slopes). Only when cultivation land can no longer be cultivated, the whole tribe will move to a new forest corner in Thanh Hoa province ; they usually gathered near Muong Lat area, in Luc Canh and Huu Thuy cantons of Quan Hoa moutain district [1].

According to the documentary named Thanh Hoa by Charles Robequain (1929), it says that Hmong people *live behind* to the Yao people (means that Hmong people live in higher areas than Yao people do) on the mountain of Phu Nghi range (in the writer's opinion Phu Nghi is now called Pu Nhi). This mountain block is at the border between Houaphanh province (Laos) and Thanh Hoa province (Vietnam), between the Ma river and Luong river, with the average high range about 1,200m. Inside Indochina, it is rarely to see the Hmong people live below the 800m altitude, they usually live on the mountain slopes with the high range between 800m and 1000m. In other words, they "jumped" from one mountain to the others despite of the administrative boundaries.

Also according to the survey study of Ch. Robequain (1929), the first Hmong people entered Muong Lat around 1900 - 1905. They live separately on high and steep slopes, because they claim that they cannot be in a hot and toxic valley, in the land of leeches and buffaloes, the dead must be buried under water, and the opium poppy cannot grow well. Hmong people have an idiom : 'the fish live in the water, the birds fly in the sky, Hmong people live in the moutain'. And also according to Ch. Robequain, he says that Hmong people do not want to admit that all the valleys have the owners (Thai and Muong people), therefore they cannot be there [3].

According to the Hmong people in Pu Nhi, Muong Lat district narrated, their ancestors migrated from Huoaphan (Laos) to Muong Lat (Thanh Hoa) about 150 years ago (6-7 generations). However, other legends tell that the Hmong people in Muong Lat and in Son La are brothers. Due to the poor circumstances, they were separated. The older brother went to Son La to bring his father's sword; and the younger brother took a cap to Laos, then he stayed at Muong Lat [1].

Thus, although it is only a legend about the family, it provides us with information that the nearest ancestors of the Hmong people in Pu Nhi (Muong Lat) came from Laos. However, it does not exclude the possibility that their previous ancestors were one of the groups who had been to the northern mountains, following the ways from China to Ha Giang or Lao Cai, and then moved everywhere and some of them went to the mountainous area in Thanh Hoa [6].

Mac Duong (1964) indicates that Miao people *(người Mèo)* living in mountainous area of Thanh- Nghe mainly resided in Muong Xen district (Nghe An) and in Quan Hoa district (Thanh Hoa). They were form Upper Laos and northwest Vietnam and migrated to these metioned province about 200 years ago [4]. From these meterials, it can be said that Hmong people in Thanh Hoa migrated directly from Laos and before from northwest Vietnam.

Consequently, Hmong people (including Yao people and Kho-mu people) entered Thanh Hoa when Thai people have already resided in low valleys that are suitable for cultivation of wet rice. To avoid to be tenants or serfs of other ethnic groups, Hmong people settled in higher mountainside. Each village has about 15 - 20 rooftops and the village is far away from the other as a space of a mountain, therefore travelling is extremely difficult.

In terms of population, in the first decades of the 20th century, the documentary related to ethnic minorities in the mountainous areas of Thanh Hoa were almost absent especially related to Hmong people. According to the earliest record of French people about this territory of H. Le Breton or Ch. Robequain also did not mention this issue either. As H. Le Breton (1918) explains that because of shifting cultivation, the Hmong people usually residing on the peaks of the mountain, hence, it is very difficult to know exactly the number of their population [1]. However, from the 1990s onwards, the Hmong migrated freely to the western region of Thanh Hoa with the large number of people made the local population increased dramatically. If before the year 1990, only 3,638 Mong people lived in the mountainous area of Thanh Hoa, 10 years later (in 1999), the number of Hmong people was up to 13,325 people [7], an increase of approximately 10,000 people, mainly from the northern mountainous provinces of Vietnam.

Currently, the Hmong people in Thanh Hoa reside mainly in two communes, Pu Nhi and Quang Chieu, Muong Lat district, but the majority are in Pu Nhi commune, basically Hmong Trang people (Mongz Dou). This is the first place that Hmong people set their foot on the land (the writer called these people are the old Hmong people). According to statistic, until 31.07.2007, there were 14,755 Hmong people in Muong Lat (accounting for 89.05% of Mong people population in Thanh Hoa). In addition, Hmong people also settle in Quan Son districts (915 people, accounting for 6.20%) and Quan Hoa district (700 people, accounting for 4.75%). In recent years, it can be seen that the picture of distribution of Hmong ethnic group in Thanh Hoa is very dynamic with a high population growth rate. According to Vietnamese Cencus in 1960, Hmong community in Thanh Hoa was 1,235 people; in 1979 there were 2,606 people; the period from 1960 to 1979 increased by 1,371 people; by 1989 there were 3,638 people; by 1999, the Hmong population in Thanh Hoa had a sudden increase by 13,325 people, the period 1989 - 1999 increased by 9,687 people, 10 times higher compared to the previous period. From 1999 to 2009 Hmong population increased by 1,097 people.

Table 1. *Hmong population in Thanh Hoa*

No.	District	Number of people
1	Muong Lat	12.789
2	Quan Hoa	708
3	Quan Son	925
Total		14.422

Source: Departments of Ethnicity, Thanh Hoa province (Statistics in 2009)

Currently, Hmong people in Thanh Hoa reside in 46 villages, including 9 communes of three border highland districts including: Muong Lat (41 villages, 5 communes), Quan Son (3 villages, 2 communes) and Quan Hoa (2 villages, 2 communes, in Buoc Hien village in Trung Thanh commune only 2 households). Muong Lat district has 3 communes in which Hmong people make up over 70% of the population (Phu Nhi commune has 13/14 villages with 807 households, 4,674 people; Trung Ly commune has 12/16 villages with 521 households, 521 households, Muong Ly commune has 11/14 villages with 464 households, 3,062 people, and Tam Chung, Quang Chieu (Muong Lat) communes; Na Mong, Son Thuy (Quan Son); Phu Son and Trung Thanh (Quan Hoa) are 2 villages where Hmong people live alternately with other ethnic groups [6].

Hmong people at the frontiers in Muong Lat include 2 communes of Pu Nhi and Nhi Son. In Pu Nhi commune, Hmong people concentrated in the villages of Pu Ngua, Hua Pu, Ban Com, Ca Noi, Ca Tau, Pha Den, Na Tao, Pu Tong and Loc Ha; in Nhi Son commune includes the villages: Cat, Pa Hoc, Chim, Keo Hu, Keo Te. Additionally, in Quan Son district, Hmong people live in the border communes of Son Thuy and Na Meo. In Son Thuy commune, Hmong people live in Xia Noi and Spring villages; in Na Meo commune, they reside in Ché Lau village.

Particularly, Hmong people in Quan Son district originated directly from Pu Nhi commune, the old Quan Hoa district. Recently, it is called Pu Nhi commune belonging to Muong Lat district. They moved to Quan Son in late 1989 (when Quan Hoa district was not separated into 3 districts: Quan Hoa, Quan Son and Muong Lat), at that time, they had only 34 households, with nearly 600 people, lived in 3 villages chosen by themselves is Xia Noi, Mua Xuan and Che Lau of Son Thuy commune, each village had more than 10 households [3]. In 1999, to implement the Decree 65 of the Government, Son Thuy commune was split into two communes: Son Thuy and Na Meo. Cho Lau village belongs to Na Meo commune.

In general, Hmong people in Thanh Hoa mostly migrated from other places in the northern mountainous region through 2 direct and indirect roads. Hmong people in Thanh Hoa have three branches: Hmong Trang (Mongz Dou), Hmong Hoa (Mongz Lenhs), Hmong Den (Mongz Duz). In which, Hmong Trang people (Mongz Dou) have the largest population and the Hmong Den (Mongz Duz) people have the smallest population. The Hmong people in Thanh Hoa have many families such as Ho, Thao, Lau, Va, Cha, Ly, Vu, Vang, and Dang. The most populous families are Lau, Ho and Thao. Due to living in high mountainous areas, their livelihood mainly is on shifting cultivation activities combined with raising, gardening and forest exploitation. Basically, Hmong ethnic community in Thanh Hoa preserve many traditional cultures, contributing to enriching the diversity of the muti-colour cultural picture of Xu Thanh ethnic communities.

3. Cultural and social characteristics of Hmong in Thanh Hoa

Like the Hmong people living in our northwest region, the resident unit of the Hmong people in Thanh Hoa likes the village of Kinh and Thai people. Hmong Trang people (Mongz Dou) call their village as *lú ro* and Hmong Den (Mongz Duz) people called it as *lu rò*. However, because of living alongside with Thai people for a long time, Hmong people often call their resident units as same as Thai people do, namely Ché Lau, Xia Non, Mua Xuan, Pu Ngua, and Lo Ha villages...

Each village of the Hmong usually has more than 10 rooftops in one place or in many places. The characteristic of the Hmong's village is that they have their own territory and business areas; the residents of a village consist of many family names and there is one more crowed family than others; each village has a common Earth Gods (head of the territory) and common conventions related to agricultural production, animal husbandry, forest protection and mutual help.

Each village of Hmong people is seen as a cultural community. Hmong people have belief in worshiping the Earth Gods, the Earth Gods may be general Gods, and sometimes be the men who exploit this land, protect and build a village or the leaders who have the duty to protect the Hmong ethnicity. In the spiritual life, early spring festivals are the chances for the whole community, especially young men and women, singing and playing, organizing folk games such as swinging, Ferris wheel, wrestling and throwing ball. In addition, there are traditional Tet ceremonies, such as Sao Nong ceremony which is held annually in January to worship in the Earth Gods (*Thổ ti, Thổ địa*).

Hmong people have a number of family names, each family name has many descents and each descent has different lines. Specially, the cohesion of family names of Hmong people is solid expressed through the authority of the Head of families, who can decide the settlement of his villages as Hmong's idiom often says that: *He holds his hand, his villagers stay; He opens his hand, his villagers leave*. People who have the same family names like Lau, Thao, Ho consider each other as brothers despite they have common old ancestor. The families in the same line do not need to reside near each other. Although they live far away, everytime they meet, they are considered as family members. Therefore, it is possible to explain why Hmong people often migrate in the lineages, even by the village [8]. This indicates the durable characteristics of Hmong's culture.

Hmong family is a patriarchal family, highly appreciating ancestral origin. As a result, family and descent relationship of the Hmong people are connected and strong, as well as they are responsible for protecting each other. That is why in married life, despite of the distance between ten generations, a man and a woman are not able to marry because of the same ancestor and blood relatives. Due to the nature of the nuclear patriarchal family (a couple and their children living together), when their son get married, he tends to live separately. That trend is expressed in the proverb of the Hmong people: "Big trees split many branches, crowed people split into small group" (*Cây to cây chia nhánh; người đông người chia nhỏ*).

Although in traditional society, Hmong's men are valued in the family, the role of women is very important and just behind the head of the family. Particularly, when the house owner decides some important issues, he discusses only with his wife and when only his wife agrees, everything go smoothly. In addition, the family is seen as a social unit, the division of labor among family members of Hmong people is also strict according to gender and age. The men often take on the heavy work such as: plowing, harrowing, cutting trees, hoeing, planting. Otherwise, the women take part in farming, doing embroidery at home, doing housework, weaving and knitting.

In Hmong's society, people always show their high respect to the elderly. Therefore, the authority and responsibility of the village patriarch is vital. If there is any conflicts in the family and village, the village patriarch is responsible for reconciliation, punishment, and being an advisor to deal with the big issues of the community like a marriage, funerals, etc.

In the ways of behavior, the Hmong in Thanh Hoa believe that when visitors come, the owners must greet the guests first, this shows the humility and respect for the guests. In communicating with the outside world, Hmong people also know how to use Thai language - like 'popular language' in mountainous areas in the west of Thanh Hoa. Currently, Hmong's young generation can speak 3 languages: mother tongue (Hmong language), Thai language and national language (Vietnamese- Kinh). Therefore, Hmong people are able to communicate with people living in Laos' border area, because most of them are mostly Thai and Hmong people. Hence, for a long time, the Hmong and Thai people in the border areas of Vietnam and Laos have a mutual understanding in exchange goods and culture. These exchanges create a strong relation between two countries day by day.

Furthermore, Hmong people have a typical physical cultural life, creating their own values, expressing ethnic identity and ethnic consciousness. It is tradition of living in house builded on the ground, stalling, covering around the house with wooden planks or bamboo wattle, and the palm-leaf roofs or embankments, or wood called *sa mu*.

Regarding to traditional clothes, Hmong's men wear loose-fitting trousers, drawstring belts. Their's clothes are usually black with some patterns, but loose-fitting trousers have no pattern at all. Additionally, Hmong women wear skirts and patterned shirts. The skirt has two types, a layer and multi-layer skirts. Hmong women's costumes in Thanh Hoa have three different designs, showing the suitability for physique, psychology and three years of age: unmarried girls, middle-aged women, and elderly women. Furthermore, women wear a turban on their head, and the men often wear soft hats or not.

In term of cuisine, Hmong people eat mainly rice, sticky rice is only available during festivals. Previously, due to the difficult conditions, Hmong people often use water instead of soup to ea with rice, or boiled vegetables eat with fish sauce or salt, boiled vegetable water is used to eat with rice. The dish of pork impregnated with salt hung in the kitchen corner for a year is also a unique dish of the Hmong people. Black meat chicken is also a special food of the Hmong people, often cooked to treat precious guests, this type of chicken is also used to make medicine for the good health.

Hmong people cerebrate the Tet's holiday following the New Year calendar, at the end of the year, in December. The time of spring celebrations usually takes three to five days, but sometimes lasts up to half of month depending on economic conditions of each village, each year. In a few days off, Hmong people prepare meals and often invite people in the same family names come to join with them for fun and joy. In those days, the households often slaughter pigs, they have the pork dishes with enough intestine of pigs, with a bottle of wine, the sticky rice, then they set on the ancestor altar (*đa xử ca*) to invite the ancestors to come back home and join the Tet's holiday with them. On the holidays and New Year, people wish each other beautiful words, hope that everyone will be healthy, have many descendants and hope for productive crops. The visitors come to Hmong village on this occasion will be warmly treated by their compatriots, which is the hospitable tradition of the Hmong people.

Like Hmong people in the northern mountainous region of our country, the Hmong people in Thanh Hoa have a rich cultural and spiritual life. First of all, the treasure of folk arts and literature include stories, folk songs, proverbs which reflects the creative ability of the masses and their awareness of nature, society, culture and national history [8].

Many folk tales reflect the unpleasant side of society: the suffering of the orphans, the grim treatment of sister-in-law with the bride, the evil step-mothers, the cruel mandarins; otherwise dignifying talented people from the residents, beautiful love stories, victors who against cruel men. The explanations about the phenomena of the physical and spiritual life of Hmong people are various (making ghosts, worshiping the women *(cúng mụ)*, women are not allowed to go upstairs, worship the columns of house, panpine dance *(múa Khèn)…*that is all remembered by family members as family activities in the normal life.

In the folk literature of Hmong people, folk songs occupy a significant position. Many folk songs have good ideological content, subtle and discreet expressions, are reminded by images closed to daily life. The common feature of folk songs is that it not only sings with lyrics but also sing with other instruments like wit pan- pine (*khèn*), Jew's harp (*đàn môi*), and leaf horns (*kèn lá*). These instruments are closely associated with folk songs of the Hmong. When people listen to pan- pine, Jew's harp and leaf horns, they deeply understand the content that the performer expresses. In folk songs, there are not only short stories but also some famous long episodes such as *Tiếng hát làm dâu* (the voice of bride), which is known by other ethnic groups. Mostly, Hmong people knows more or less the folk songs as well as the way of using their traditional instruments.

Hmong's folk music is lyrical and plentiful which reflects the natural beauty of the highlands, the bright beauty of Hmong's spirits, therefore many researchers are interested in studying Hmong's music.

Hmong's musical instruments are very unique. Panpine is used in funerals and some family rituals. At the time when the men go for walk or relax under the moon at night after a long hardworking day, they tend to blow panpines. Also, Jew's harp and learf horns are the means of exchanging the hearts of young men and women in the late night. Especially, the meaning behinds the word of the song they play with those instruments is more powerful than the word itself [5]. Moreover, for Hmong people in Thanh Hoa in general, flute is also a close and popular instrument in their spiritual life. This is also an instrument that is easy to express heart, especially the feelings of boys and girls in love.

In some areas of Thanh Hoa province, the panpine dances of Hmong people are also popular with Kinh, Muong, and Thai people. They use panpine dances in parties, events, public holidays, public celebrations, house celebrations, New Year celebrations. In some special event like the music events to welcome the Congress, welcome the honour guests, Hmong's panpipe dances is performed by Hmong people or by other ethnic groups.

However, cultural and spiritual values including the treasure of folklore, music of Hmong people has been eroded. It does only exist in the older generations, for the younger generation, few of them is interested in and has little understanding about it.

4. Conclusion

To sum up, Hmong people in Vietnam ingeneral and in Thanh Hoa in particular have the late migration and settlement processes compared to other ethnic groups in our country. So far, there has been a little change in Hmong communities in term of population. In the past and until now, Hmong people mainly take up shifting cultivation and animal husbandry as their main economic activities; other economic activities are only complementary. Additionally, Hmong people have a high sense of community, spirit of mutual support to help each other when the village have to deal with great work. They also have a sense of material contribution to spiritual and cultural activities for community events Therefore, the values of the history, culture and society cultural are the solid base for Hmong people to survive and develop sustainably in their own homeland.

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EXPONENTIAL STABILITY OF 2D DISCRETE SYSTEMS WITH MIXED TIME-VARYING DELAYS

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Received: 30 March 2018/ Accepted: 11 June 2019/ Published: June 2019

Abstract: *This paper deals with the problem of exponential stability of two-dimensional (2D) discrete-time systems with mixed directional time-varying delays. By constructing an improved 2D Lyapunov-Krasovskii functional candidate some new delay-dependent condition for the exponential stability of the system are derived in terms of linear matrix inequalities (LMIs).*

Keywords: 2D systems, Exponential stability, Lyapunov-Krasovskii function, Linear matrix inequalities (LMIs).

1. Introduction

Two-dimensional systems have many applications in different arears as geographical data processing, electrical circuit networks, power systems, energy exchange processes, multibody mechanics, process control, aerospace engineering and physical processes [18, 4, 9, 14]. In recent years, 2-D switched systems have attracted the attention of various scientists who have made the significant contributions in stability theory. Most commonly utilized state-space models of 2D systems are the Roesser model, the Fornasini-Marchesini (FM) local model and the Attasi model [18, 17, 5, 4]. Time-delay phenomena are frequently in various practical systems. The existence of time delay may lead to instability or poor performance of the system, so it is of significance to study time-delay systems. The exponential stability for 2D state delay systems has been studied. There have been many previous results on stability for 2D discrete systems with time-varying delays[3, 13, 6, 7, 12, 19]. However, to the best of our knowledge, the problem of stability 2D systems with state delays, especially for 2D systems with mixed delays, has not been fully investigated to date.

In this paper, we study the problem of exponential stability of a class of 2D discrete-time systems described by the Roesser model with mixed time-varying delays. Delay-range-dependent exponential stability criteria of 2D systems discrete-time with mixed time-varying delays are established in terms of linear matrix inequalities .

Notations: Z denotes the set of integers, $Z[a,b] \triangleq \{a, a+1, \ldots, b\}$ for $a, b \in Z$, $a \le b$. $R^{n\times m}$ denotes the set of $n\times m$ real matrices and $\mathrm{diag}(A,B) \triangleq \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$ for two matrices A, B. $\mathrm{Sym}(A) \triangleq A + A^{\mathrm{T}}$ for $A \in R^{n\times n}$. A matrix $M \in R^{n\times n}$ is semi-positive definite, $M \ge 0$, if $x^{\mathrm{T}} Mx \ge 0$, $\forall x \in R^n$; M is positive definite, $M > 0$, if $x^{\mathrm{T}} Mx > 0$, $\forall x \in R^n$, $x \ne 0$.

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2. Preliminaries

Consider a class of 2-D discrete-time systems with mixed directional time-varying delays described by the following Roesser model (2-D DRM)

$$\begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix} = A\begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} + A_\tau \begin{bmatrix} x^h(i-\tau_h(i),j) \\ x^v(i,j-\tau_v(j)) \end{bmatrix} + A_d \begin{bmatrix} \sum_{l=1}^{d_h(i)} x^h(i-l,j) \\ \sum_{t=1}^{d_v(j)} x^v(i,j-t) \end{bmatrix}, \quad i,j \in Z^+, \tag{1}$$

where $x^h(i,j) \in R^{n_h}$ and $x^v(i,j) \in R^{n_v}$ are the horizontal state vector and the vertical state vector, respectively. A, A_τ and A_d are constant matrices with appropriate dimensions. $\tau_h(i)$, $d_h(i)$ and $\tau_v(j)$, $d_v(j)$ are respectively the directional time-varying delays along the horizontal and vertical directions satisfying

$$\tau_{hm} \le \tau_h(i) \le \tau_{hM}, \quad \tau_{vm} \le \tau_v(j) \le \tau_{vM}, \tag{2}$$

$$d_{hm} \le d_h(i) \le d_{hM}, \quad d_{vm} \le d_v(j) \le d_{vM}, \tag{3}$$

where τ_{hm}, τ_{hM}, τ_{vm}, τ_{vM}, d_{hm}, d_{hM}, d_{vm} and d_{vM} are known nonnegative integers involving the upper and the lower bounds of delays. Denote $\mu_h = \max(\tau_{hM}, d_{hM})$ and $\mu_v = \max(\tau_{vM}, d_{vM})$. Initial condition of (1) is defined by

$$\begin{aligned} x^h(i,j) &= \phi(i,j), i \in Z[-\mu_h, 0], 0 \le j \le z_1, \\ x^h(i,j) &= 0, j > z_1 \\ x^v(i,j) &= \psi(i,j), j \in Z[-\mu_v, 0], 0 \le i \le z_2, \\ x^v(i,j) &= 0, i > z_2, \end{aligned} \tag{4}$$

where $\phi(k,.) \in l_2(Z^+), \forall k \in Z[-\mu_h, 0]$ and $\psi(.,l) \in l_2(Z^+), \forall l \in Z[-\mu_v, 0]$, $z_1 < \infty$ and $z_2 < \infty$.

Definition 1. *System (1) is said to be exponentially stable if there exist scalars $N > 0$ and $0 < \beta < 10$ such that any solution $x(i,j)$ of (1) satisfies*

$$\sum_{i+j=\Gamma} \| x(i,j) \|^2 \le N\beta^{(\Gamma-\kappa_0)} \sum_{i+j=\kappa_0} \| x(i,j) \|_C^2 \tag{5}$$

holds for all $i + j = \Gamma \ge \kappa_0 = i_\kappa + j_\kappa$, where

$$\sum_{i+j=\Gamma} \| x(i,j) \|_C^2 \triangleq \sup_{\substack{-\mu_v \le t \le 0 \\ -\mu_h \le s \le 0}} \sum_{i+j=\kappa_0} \{\| x^h(i+s,j) \|^2 + \| x^v(i,j+t) \|^2, \| z^h(i+s,j) \|^2 + \| z^v(i,j+t) \|^2\},$$

$$z^h(i+s,j) = x^h(i+s+1,j) - x^h(i+s,j), \; z^v(i,j+t) = x^v(i,j+t+1) - x^v(i,j+t).$$

Lemma 1. [3] *For any vector $\omega(t) \in R^n$, two positive integers ℓ_1 and ℓ_2, and a symmetric positive matrix $H \in R^{n \times n}$, the following inequality holds,*

$$-(\ell_2-\ell_1-1)\sum_{t=\ell_2}^{\ell_1}\omega^{\mathrm{T}}(t)H\omega(t)\le\left[\sum_{t=\ell_2}^{\ell_1}\omega^{\mathrm{T}}(t)\right]H\left[\sum_{t=\ell_2}^{\ell_1}\omega(t)\right] \tag{6}$$

Lemma 2. [3] *For a symmetric positive definite matrix* $R\in R^{n\times n}$, *positive integers* h,v *and a function* $x:Z[i-h,i]\times[j-v,j]\to R^n$, $i,j\in Z^+$, *the following inequalities hold*

$$\sum_{l=i-h}^{i-1}\delta_1^{\mathrm{T}}(l,j)R\delta_1(l,j)\ge\frac{1}{h}\left[x(i,j)-x(i-h,j)\right]^{\mathrm{T}}R\left[x(i,j)-x(i-h,j)\right], \tag{7}$$

$$\sum_{s=j-v}^{j-1}\delta_2^{\mathrm{T}}(i,s)R\delta_2(i,s)\ge\frac{1}{v}\left[x(i,j)-x(i,j-v)\right]^{\mathrm{T}}R\left[x(i,j)-x(i,j-v)\right], \tag{8}$$

where $\delta_1(l,j)=x(l+1,j)-x(l,j)$ and $\delta_2(i,s)=x(i,s+1)-x(i,s)$.

3. Main results

We are now in a position to derive LMI-based conditions ensuring that system (1) exponential stable. For the brevity, in the following we denote the block matrix $I(\alpha,\beta)=diag(\alpha I_{n_h},\beta I_{n_v})$ for any scalars α,β.

Theorem 1. *For given nonnegative integers* τ_{hm},τ_{hM}, τ_{vm},τ_{vM}, d_{hm},d_{hM}, d_{vm} *and* d_{vM}, *if there exist symmetric positive definite matrices* $P=diag(P_h,P_v)$, $Q=diag(Q_h,Q_v)$, $R=diag(R_h,R_v)$, $X=diag(X_h,X_v)$, $Y=diag(Y_h,Y_v)$, $S=diag(S_h,S_v)$, $Z=diag(Z_h,Z_v)$ *and* $0<\beta<1$ *such that the following LMI holds*

$$\Phi=\begin{bmatrix}\Psi & A^{\mathrm{T}}P & D^{\mathrm{T}}\Upsilon\\ & -P & 0\\ & * & -\Pi\end{bmatrix}<0 \tag{9}$$

where $\Upsilon=\begin{bmatrix}X & Y & S\end{bmatrix}$, $\Pi=diag(X,Y,S)$, and

$$\Psi=\begin{bmatrix}\Psi_{11} & 0 & X & \overline{Y} & 0\\ & -(\overline{R}+2\overline{S}) & \overline{S} & \overline{S} & 0\\ & * & -(Q+X+\overline{S}) & 0 & 0\\ & * & * & -(\overline{X}+\overline{S}) & 0\\ & * & * & * & -Z\end{bmatrix},$$

$$\Psi_{11}=\overline{Q}+R+Z-P-X-\overline{Y}$$

$$X=I(\beta\tau_{hm}^2,\beta\tau_{vm}^2)X,\quad Y=I(\beta\tau_{hM}^2,\beta\tau_{vM}^2)Y,\quad S=I(\beta r_{\tau h}^2,\beta r_{\tau v}^2)S,$$

$$R=I(\beta(1+r_{\tau h}),\beta(1+r_{\tau v}))R,\quad Z=I(\beta r_{dh},\beta r_{dv})Z,\quad P=I(\beta,\beta)P,\overline{Q}=I(\beta,\beta)Q,$$

$$X = I(\beta^{1+\tau_{hm}}, \beta^{1+\tau_{vm}})X, \quad \overline{Y} = I(\beta^{1+\tau_{hM}}, \beta^{1+\tau_{vM}})Y, \quad \overline{R} = I(\beta^{1+\tau_{hM}}, \beta^{1+\tau_{vM}})R,$$

$$\overline{S} = I(\beta^{1+\tau_{hM}}, \beta^{1+\tau_{vM}})S, \quad Q = I(\beta^{1+\tau_{hm}}, \beta^{1+\tau_{vm}})Q, \quad \overline{X} = I(\beta^{1+\tau_{hM}}, \beta^{1+\tau_{vM}})X,$$

$$r_{\tau h} = \tau_{hM} - \tau_{hm}, \quad r_{dh} = \frac{d_{hM}(d_{hM} + d_{hm})(d_{hM} - d_{hm} + 1)}{2},$$

$$r_{\tau v} = \tau_{vM} - \tau_{vm}, \quad r_{dv} = \frac{d_{vM}(d_{vM} + d_{vm})(d_{vM} - d_{vm} + 1)}{2},$$

$$A = \begin{bmatrix} A & A_\tau & 0 & 0 & A_d \end{bmatrix}, \quad D = \begin{bmatrix} A - I & A_\tau & 0 & 0 & A_d \end{bmatrix},$$

then system (1) is exponentially stable.

Proof. For the brevity, in the following, we denote

$$x(i,j) = \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix}, \quad x(i+1, j+1) = \begin{bmatrix} x^h(i+1, j) \\ x^v(i, j+1) \end{bmatrix},$$

$$x_\tau(i,j) = \begin{bmatrix} x^h(i - \tau_h(i), j) \\ x^v(i, j - \tau_v(j)) \end{bmatrix}, \quad x_{\tau M}(i,j) = \begin{bmatrix} x^h(i - \tau_{hM}), j) \\ x^v(i, j - \tau_{vM}) \end{bmatrix},$$

$$x_{\tau m}(i,j) = \begin{bmatrix} x^h(i - \tau_{hm}), j) \\ x^v(i, j - \tau_{vm}) \end{bmatrix}, \quad x_d(i,j) = \begin{bmatrix} \sum_{l=1}^{d_h(i)} x^h(i-l, j) \\ \sum_{t=1}^{d_v(j)} x^v(i, j-t) \end{bmatrix},$$

$$\delta^h(i,j) = x^h(i+1, j) - x^h(i,j), \quad \delta^v(i,j) = x^v(i, j+1) - x^v(i,j),$$

$$\eta(i,j) = \begin{bmatrix} x^{\mathrm{T}}(i,j) & x_\tau^{\mathrm{T}}(i,j) & x_{\tau m}^{\mathrm{T}}(i,j) & x_{\tau M}^{\mathrm{T}}(i,j) & x_d^{\mathrm{T}}(i,j) \end{bmatrix}^{\mathrm{T}}.$$

We consider the following Lyapunov-Krasovskii functional

$$V(i,j) = \underbrace{\sum_{q=1}^{8} V_q^h(x^h(i,j))}_{V^h(i,j)} + \underbrace{\sum_{q=1}^{8} V_q^v(x^v(i,j))}_{V^v(i,j)} \tag{10}$$

where $V_1^h(x^h(i,j)) = x^{h\mathrm{T}}(i,j) P_h x^h(i,j),$

$$V_2^h(x^h(i,j)) = \sum_{l=i-\tau_{hm}}^{i-1} x^{h\mathrm{T}}(l,j) Q_h x^h(l,j) \beta^{i-l},$$

$$V_3^h(x^h(i,j)) = \sum_{l=i-\tau_h(i)}^{i-1} x^{h\mathrm{T}}(l,j) R_h x^h(l,j) \beta^{i-l},$$

$$V_4^h(x^h(i,j)) = \sum_{s=-\tau_{hM}+1}^{-\tau_{hm}} \sum_{l=i+s}^{i-1} x^{h\mathrm{T}}(l,j) R_h x^h(l,j) \beta^{i-l},$$

$$V_5^h(x^h(i,j)) = \tau_{hm}\sum_{s=-\tau_{hm}}^{-1}\sum_{l=i+s}^{i-1}\delta^{h\mathrm{T}}(l,j)X_h\delta^h(l,j)\beta^{i-l},$$

$$V_6^h(x^h(i,j)) = \tau_{hM}\sum_{s=-\tau_{hM}}^{-1}\sum_{l=i+s}^{i-1}\delta^h(l,j)^T Y_h\delta^h(l,j)\beta^{i-l},$$

$$V_7^h(x^h(i,j)) = r_{\tau h}\sum_{s=-\tau_{hM}}^{-\tau_{hm}-1}\sum_{l=i+s}^{i-1}\delta^{h\mathrm{T}}(l,j)S_h\delta^h(l,j)\beta^{i-l},$$

$$V_8^h(x^h(i,j)) = d_{hM}\sum_{s=d_{hm}}^{d_{hM}}\sum_{l=1}^{s}\sum_{p=i-l}^{i-1}x^{h\mathrm{T}}(p,j)Z_h x^h(p,j)\beta^{i-p},$$

and

$$V_1^v(x^v(i,j)) = x^{v\mathrm{T}}(i,j)P_v x^v(i,j),$$

$$V_2^v(x^v(i,j)) = \sum_{t=j-\tau_{vm}}^{j-1}x^{v\mathrm{T}}(i,t)Q_v x^v(i,t)\beta^{j-t},$$

$$V_3^v(x^v(i,j)) = \sum_{t=j-\tau_v(j)}^{j-1}x^{v\mathrm{T}}(i,t)R_v x^v(i,t)\beta^{j-t},$$

$$V_4^v(x^v(i,j)) = \sum_{k=-\tau_{vM}+1}^{-\tau_{vm}}\sum_{t=j+k}^{j-1}x^{v\mathrm{T}}(i,t)R_v x^v(i,t)\beta^{j-t},$$

$$V_5^v(x^v(i,j)) = \tau_{vm}\sum_{k=-\tau_{vm}}^{-1}\sum_{t=j+k}^{j-1}\delta^{v\mathrm{T}}(i,t)X_v\delta^v(i,t)\beta^{j-t},$$

$$V_6^v(x^v(i,j)) = \tau_{vM}\sum_{k=-\tau_{vM}}^{-1}\sum_{t=j+k}^{j-1}\delta^{v\mathrm{T}}(i,t)Y_v\delta^v(i,t)\beta^{j-t},$$

$$V_7^v(x^h(i,j)) = r_{\tau v}\sum_{k=-\tau_{vM}}^{-\tau_{vm}-1}\sum_{t=j+k}^{j-1}\delta^{v\mathrm{T}}(i,t)S_v\delta^v(i,t)\beta^{j-t},$$

$$V_8^v(x^v(i,j)) = d_{vM}\sum_{k=d_{vm}}^{d_{vM}}\sum_{t=1}^{k}\sum_{p=j-t}^{j-1}x^{v\mathrm{T}}(i,p)Z_v x^v(i,p)\beta^{j-p}.$$

Clearly, $V(i,j)\ge 0, \forall i,j\in Z^+$. With respect to 2-D DRM (1), the $\Delta V_\beta(i,j)$ is defined directionally as follows

$$\Delta V_\beta(i,j) \triangleq V^h(i+1,j)-\beta V^h(i,j)+V^v(i,j+1)-\beta V^v(i,j)\triangleq \Delta V_\beta^h(i,j)+\Delta V_\beta^v(i,j) \quad (11)$$

First, we have

$$\Delta V_{1\beta}^h(x^h(i,j)) = x^{h\mathrm{T}}(i+1,j)P_h x(i+1,j)-\beta x^{h\mathrm{T}}(i,j)P_h x^h(i,j)$$

$$\Delta V_{2\beta}^h((x^h(i,j)) = \beta x^{h\mathrm{T}}(i,j)Q_h x(i,j)-\beta^{1+\tau_{hm}}x^{h\mathrm{T}}(i-\tau_{hm},j)Q_h x^h(i-\tau_{hm},j)$$

$$\Delta V_{3\beta}^h((x^h(i,j)) = \sum_{l=i+1-\tau_h(i+1)}^{i}x^{h\mathrm{T}}(l,j)R_h x^h(l,j)-\sum_{l=i-\tau_h(i)}^{i-1}x^{h\mathrm{T}}(l,j)R_h x^h(l,j)$$

$$\leq \beta x^{hT}(i,j)R_h x^h(i,j) - \beta^{1+\tau_{hM}} x^{hT}(i-\tau_h(i),j)R_h x^h(i-\tau_h(i),j) + \sum_{l=i+1-\tau_{hM}}^{i-\tau_{hm}} x^{hT}(l,j)R_h x^h(l,j)\beta^{i+1-l}$$

$$\Delta V_{4\beta}^h((x^h(i,j)) = \sum_{s=-\tau_{hM}}^{-\tau_{hm}} \left[\sum_{l=i+1+s}^{i} x^{hT}(l,j)R_h x^h(l,j)\beta^{i+1-l} - \sum_{l=i+s}^{i-1} x^{hT}(l,j)R_h x^h(l,j)\beta^{i+1-l} \right]$$

$$\leq \sum_{s=-\tau_{hM}}^{-\tau_{hm}} \left[\beta x^{hT}(i,j)R_h x(i,j) - x^{hT}(i+s,j)R_h x(i+s,j)\beta^{1-s} \right]$$

$$\leq \left[r_{\tau h}\beta x^{hT}(i,j)R_h x(i,j) - \sum_{l=i+1-\tau_{hM}}^{i-\tau_{hm}} x^{hT}(l,j)R_h x^h(l,j)\beta^{i+1-l} \right] \tag{12}$$

and $\Delta V_{n\beta}^h((x^h(i,j))$ ($n = 5,6,7$) are given as

$$\begin{aligned}
\Delta V_{5\beta}^h(x^h(i,j)) &= \tau_{hm} \sum_{s=-\tau_{hm}}^{-1} \left[\sum_{l=i+1+s}^{i} \delta^{hT}(l,j)X_h\delta^h(l,j)\beta^{i+1-l} - \sum_{l=i+s}^{i-1} \delta^{hT}(l,j)X_h\delta^h(l,j)\beta^{i+1-l} \right] \\
&\leq \tau_{hm}^2 \beta\delta^{hT}(i,j)X_h\delta^h(i,j) - \tau_{hm}\beta^{1+\tau_{hm}} \sum_{l=i-\tau_{hm}}^{i-1} \delta^{hT}(l,j)X_h\delta^h(l,j), \\
\Delta V_{6\beta}^h(x^h(i,j)) &= \tau_{hM} \sum_{s=-\tau_{hM}}^{-1} \left[\sum_{l=i+1+s}^{i} \delta^{hT}(l,j)Y_h\delta^h(l,j)\beta^{i+1-l} - \sum_{l=i+s}^{i-1} \delta^{hT}(l,j)Y_h\delta^h(l,j)\beta^{i+1-l} \right] \\
&\leq \tau_{hM}^2 \beta\delta^{hT}(i,j)Y_h\delta^h(i,j) - \tau_{hM}\beta^{1+\tau_{hM}} \sum_{l=i-\tau_{hM}}^{i-1} \delta^{hT}(l,j)Y_h\delta^h(l,j), \\
\Delta V_{7\beta}^h(x^h(i,j)) &= r_{\tau h} \sum_{s=-\tau_{hM}}^{\tau_{hm}-1} \left[\sum_{l=i+1+s}^{i} \delta^{hT}(l,j)S_h\delta^h(l,j)\beta^{i+1-l} - \sum_{l=i+s}^{i-1} \delta^{hT}(l,j)S_h\delta^h(l,j)\beta^{i+1-l} \right] \\
&\leq r_{\tau h}^2 \beta\delta^{hT}(i,j)S_h\delta^h(i,j) - r_{\tau h}\beta^{1+\tau_{hM}} \sum_{l=i-\tau_{hM}}^{i-\tau_{hm}-1} \delta^{hT}(l,j)S_h\delta^h(l,j)
\end{aligned} \tag{13}$$

By Lemma 2, we have

$$\begin{aligned}
-\tau_{hm}\beta^{1+\tau_{hm}} &\sum_{l=i-\tau_{hm}}^{i-1} \delta^{hT}(l,j)X_h\delta^h(l,j) \\
&\leq -[x^h(i,j) - x^h(i-\tau_{hm},j)]^T \left(\beta^{1+\tau_{hm}} X_h \right) [x^h(i,j) - x^h(i-\tau_{hm},j)] \\
&= \begin{bmatrix} x^h(i,j) \\ x^h(i-\tau_{hm},j) \end{bmatrix}^T \begin{bmatrix} -\beta^{1+\tau_{hm}} X_h & \beta^{1+\tau_{hm}} X_h \\ & -\beta^{1+\tau_{hm}} X_h \end{bmatrix} \begin{bmatrix} x^h(i,j) \\ x^h(i-\tau_{hm},j) \end{bmatrix}
\end{aligned} \tag{14}$$

and

$$-\tau_{hM}\beta^{1+\tau_{hM}}\sum_{l=i-\tau_{hM}}^{i-1}\delta^{hT}(l,j)Y_h\delta^h(l,j)$$
$$\le -[x^h(i,j)-x^h(i-\tau_{hM},j)]^T\left(\beta^{1+\tau_{hM}}Y_h\right)[x^h(i,j)-x^h(i-\tau_{hM},j)]$$
$$=\begin{bmatrix} x^h(i,j) \\ x^h(i-\tau_{hM},j)\end{bmatrix}^T\begin{bmatrix} -\beta^{1+\tau_{hM}}Y_h & \beta^{1+\tau_{hM}}Y_h \\ & -\beta^{1+\tau_{hM}}Y_h\end{bmatrix}\begin{bmatrix} x^h(i,j) \\ x^h(i-\tau_{hM},j)\end{bmatrix} \quad (15)$$

$$-r_{\tau h}\beta^{1+\tau_{hM}}\sum_{l=i-\tau_{hM}}^{i-\tau_{hm}-1}\delta^{hT}(l,j)S_h\delta^h(l,j)$$
$$\le -(\tau_h(i)-\tau_{hm})\sum_{l=i-\tau_h(i)}^{i-\tau_{hm}-1}\delta^{hT}(l,j)\beta^{1+\tau_{hM}}S_h\delta(l,j)-(\tau_{hM}-\tau_h(i))\sum_{l=i-\tau_{hM}}^{i-\tau_h(i)-1}\delta^{hT}(l,j)\beta^{1+\tau_{hM}}S_h\delta(l,j)$$
$$\le -[\sum_{l=i-\tau_h(i)}^{i-\tau_{hm}-1}\delta^{hT}(l,j)]\beta^{1+\tau_{hM}}S_h[\sum_{l=i-\tau_h(i)}^{i-\tau_{hm}-1}\delta^h(l,j)]-[\sum_{l=i-\tau_{hM}}^{i-\tau_h(i)-1}z^{hT}(l,j)]\beta^{1+\tau_{hM}}S_h[\sum_{l=i-\tau_{hM}}^{i-\tau_h(i)-1}\delta^h(l,j)]$$
$$\le \begin{bmatrix} x^h(i-\tau_h(i),j) \\ x^h(i-\tau_{hm},j) \\ x^h(i-\tau_{hM},j)\end{bmatrix}^T C_o\otimes\left(\beta^{1+\tau_{hM}}S_h\right)\begin{bmatrix} x^h(i-\tau_h(i),j) \\ x^h(i-\tau_{hm},j) \\ x^h(i-\tau_{hM},j)\end{bmatrix} \quad (16)$$

where $C_o=\begin{bmatrix} -2 & 1 & 1 \\ & -1 & 0 \\ & * & -1\end{bmatrix}$ and the symbol $\otimes$ denotes the Kronecker product of two matrices.

By Lemma 1 again, $\Delta V_{8\beta}^h((x^h(i,j))$ is given as

$$\Delta V_{8\beta}^h(x^h(i,j)) = d_{hM}\sum_{s=d_{hm}}^{d_{hM}}\sum_{l=1}^{s}(\sum_{p=i+1-l}^{i}x^{hT}(p,j)Z_hx^h(p,j)\beta^{i+1-p}-\sum_{p=i-l}^{i-1}x^{hT}(p,j)Z_hx^h(p,j)\beta^{i+1-p})$$
$$=r_{dh}\beta x^{hT}(i,j)Z_hx^h(i,j)-d_{hM}\sum_{s=d_{hm}}^{d_{hM}}\sum_{l=1}^{s}x^{hT}(i-l,j)Z_hx^h(i-l,j)\beta^{1-l}$$
$$\le r_{dh}\beta x^{hT}(i,j)Z_hx^h(i,j)-d_{hM}\sum_{l=1}^{d_h(i)}x^{hT}(i-l,j)Z_hx^h(i-l,j)$$
$$\le r_{dh}\beta x^{hT}(i,j)Z_hx^h(i,j)-d_h(i)\sum_{l=1}^{d_h(i)}x^{hT}(i-l,j)Z_hx^h(i-l,j)$$
$$\le r_{dh}\beta x^{hT}(i,j)Z_hx^h(i,j)-(\sum_{l=1}^{d_h(i)}x^h(i-l,j))^T Z_h(\sum_{l=1}^{d_h(i)}x^h(i-l,j)) \quad (17)$$

From (12) to (17) we have

$$
\begin{aligned}
\sum_{n=1}^{8}\Delta V_{n\beta}^{h}(x^{h}(i,j)) \le\; & x^{hT}(i+1,j)P_h x(i+1,j)+x^{hT}(i,j)(\beta Q_h+\beta(1+r_{\tau h})R_h+\beta r_{dh}Z_h-\beta P_h)x^{h}(i,j) \\
& -\beta^{1+\tau_{hm}}x^{hT}(i-\tau_{hm},j)Q_h x^{h}(i-\tau_{hm},j)-\beta^{1+\tau_{hM}}x^{hT}(i-\tau_h(i),j)R_h x^{h}(i-\tau_h(i),j) \\
& +\delta^{hT}(i,j)(\tau_{hm}^2\beta X_h+\tau_{hM}^2\beta Y_h+r_{\tau h}^2\beta S_h)\delta^{h}(i,j) \\
& +\begin{bmatrix} x^{h}(i,j) \\ x^{h}(i-\tau_{hm},j)\end{bmatrix}^{T}\begin{bmatrix} -\beta^{1+\tau_{hm}}X_h & \beta^{1+\tau_{hm}}X_h \\ & -\beta^{1+\tau_{hm}}X_h\end{bmatrix}\begin{bmatrix} x^{h}(i,j) \\ x^{h}(i-\tau_{hm},j)\end{bmatrix} \\
& +\begin{bmatrix} x^{h}(i,j) \\ x^{h}(i-\tau_{hM},j)\end{bmatrix}^{T}\begin{bmatrix} -\beta^{1+\tau_{hM}}Y_h & \beta^{1+\tau_{hM}}Y_h \\ & -\beta^{1+\tau_{hM}}Y_h\end{bmatrix}\begin{bmatrix} x^{h}(i,j) \\ x^{h}(i-\tau_{hM},j)\end{bmatrix} \\
& +\begin{bmatrix} x^{h}(i-\tau_h(i),j) \\ x^{h}(i-\tau_{hm},j) \\ x^{h}(i-\tau_{hM},j)\end{bmatrix}^{T} C_o\otimes\left(\beta^{1+\tau_{hM}}S_h\right)\begin{bmatrix} x^{h}(i-\tau_h(i),j) \\ x^{h}(i-\tau_{hm},j) \\ x^{h}(i-\tau_{hM},j)\end{bmatrix} \\
& -\Big(\sum_{l=1}^{d_h(i)}x^{h}(i-l,j)\Big)^{T}Z_h\Big(\sum_{l=1}^{d_h(i)}x^{h}(i-l,j)\Big) \\
\le\; & x^{hT}(i+1,j)P_h x(i+1,j)+\delta^{hT}(i,j)(\widehat{Xh+Yh+Sh})\delta^{h}(i,j)+\eta^{hT}(i,j)\Psi_h\eta^{h}(i,j) \quad (18)
\end{aligned}
$$

where $Xh=\tau_{hm}^2\beta X_h$, $Yh=\tau_{hM}^2\beta Y_h$, $Sh=r_{\tau h}^2\beta S_h$ and

$$
\eta^{h}(i,j)=\begin{bmatrix} x^{hT}(i,j) & x^{hT}(i-\tau_h(i),j) & x^{hT}(i-\tau_{hm},j) & x^{hT}(i-\tau_{hM},j) & \sum_{l=1}^{d_h(i)}x^{hT}(i-l,j)\end{bmatrix}^{T},
$$

$$
\Psi_h=\begin{bmatrix}
\Psi_{h11} & 0 & \beta^{1+\tau_{hm}}X_h & \beta^{1+\tau_{hM}}Y_h & 0 \\
 & -\beta^{1+\tau_{hM}}R_h-2\beta^{1+\tau_{hM}}S_h & \beta^{1+\tau_{hM}}S_h & \beta^{1+\tau_{hM}}S_h & 0 \\
 & * & -\beta^{1+\tau_{hm}}(Q_h+X_h)-\beta^{1+\tau_{hM}}S_h & 0 & 0 \\
 & * & * & -\beta^{1+\tau_{hM}}(X_h+S_h) & 0 \\
 & * & * & * & -Z_h
\end{bmatrix},
$$

$$
\Psi_{h11}=\beta Q_h+\beta(1+r_{\tau h})R_h+\beta r_{dh}Z_h-\beta P_h-\beta^{1+\tau_{hm}}X_h-\beta^{1+\tau_{hM}}Y_h.
$$

Similarly, we have

$$
\Delta V_{1\beta}^{v}(x^{v}(i,j))=x^{vT}(i,j+1)P_v x(i,j+1)-\beta x^{vT}(i,j)P_v x^{v}(i,j)
$$

$$
\Delta V_{2\beta}^{v}((x^{v}(i,j))=\beta x^{vT}(i,j)Q_v x(i,j)-\beta^{1+\tau_{vm}}x^{vT}(i,j-\tau_{vm})Q_v x^{v}(i,j-\tau_{vm})
$$

$$\Delta V_{3\beta}^{v}((x^{v}(i,j)) \le \beta x^{v\mathrm{T}}(i,j)R_{v}x^{v}(i,j) - \beta^{1+\tau_{vM}} x^{v\mathrm{T}}(i,j-\tau_{v}(j))R_{v}x^{v}(i,j-\tau_{v}(j)) + \sum_{t=j+1-\tau_{vM}}^{j-\tau_{vm}} x^{v\mathrm{T}}(i,t)R_{v}x^{v}(i,t)\beta^{j+1-t}$$

$$\Delta V_{4\beta}^{v}((x^{v}(i,j)) \le \left[\beta(r_{\tau v})x^{v\mathrm{T}}(i,j)R_{v}x(i,j) - \sum_{t=j+1-\tau_{vM}}^{j-\tau_{vm}} x^{v\mathrm{T}}(i,t)R_{v}x^{v}(i,t)\beta^{j+1-t}\right]$$

$$\Delta V_{5\beta}^{v}(x^{v}(i,j)) \le \tau_{vm}^{2}\beta\delta^{v\mathrm{T}}(i,j)X_{v}\delta^{v}(i,j) - \begin{bmatrix} x^{v}(i,j) \\ x^{v}(i,j-\tau_{vm}) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} -\beta^{1+\tau_{vm}}X_{v} & \beta^{1+\tau_{vm}}X_{v} \\ & -\beta^{1+\tau_{vm}}X_{v} \end{bmatrix} \begin{bmatrix} x^{v}(i,j) \\ x^{v}(i,j-\tau_{vm}) \end{bmatrix}$$

$$\Delta V_{6\beta}^{v}(x^{v}(i,j)) \le \tau_{vm}^{2}\beta\delta^{v\mathrm{T}}(i,j)X_{v}\delta^{v}(i,j) - \begin{bmatrix} x^{v}(i,j) \\ x^{v}(i,j-\tau_{vm}) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} -\beta^{1+\tau_{vm}}X_{v} & \beta^{1+\tau_{vm}}X_{v} \\ & -\beta^{1+\tau_{vm}}X_{v} \end{bmatrix} \begin{bmatrix} x^{v}(i,j) \\ x^{v}(i,j-\tau_{vm}) \end{bmatrix}$$

$$\Delta V_{7\beta}^{v}(x^{v}(i,j)) \le r_{\tau v}^{2}\beta\delta^{v\mathrm{T}}(i,j)S_{v}\delta^{v}(i,j) - \begin{bmatrix} x^{v}(i,j-\tau_{v}(j)) \\ x^{v}(i,j-\tau_{hv}) \\ x^{v}(i,j-\tau_{vM}) \end{bmatrix}^{\mathrm{T}} C_{o} \otimes \left(\beta^{1+\tau_{vM}} S_{v}\right) \begin{bmatrix} x^{v}(i,j-\tau_{v}(j)) \\ x^{v}(i,j-\tau_{vm}) \\ x^{v}(i,j-\tau_{vM}) \end{bmatrix}$$

$$\Delta V_{8\beta}^{v}(x^{v}(i,j)) \le r_{dv}\beta x^{v\mathrm{T}}(i,j)Z_{v}x^{v}(i,j) - \left(\sum_{t=1}^{d_{v}(j)} x^{v}(i,j-t)\right)^{\mathrm{T}} Z_{v}\left(\sum_{t=1}^{d_{v}(j)} x^{v}(i,j-t)\right) \quad (19)$$

Therefore

$$\sum_{n=1}^{8}\Delta V_{n\beta}^{v}(x^{v}(i,j)) \le x^{v\mathrm{T}}(i,j+1)P_{v}x(i,j+1) + \delta^{v\mathrm{T}}(i,j)(X_{v}+Y_{v}+S_{v})\delta^{v}(i,j) + \eta^{v\mathrm{T}}(i,j)\Psi_{v}\eta^{v}(i,j) \quad (20)$$

where $X_{v} = \tau_{vm}^{2}\beta X_{v}$, $Y_{v} = \tau_{vM}^{2}\beta Y_{v}$, $S_{v} = r_{\tau v}^{2}\beta S_{v}$ and

$$\eta^{v}(i,j) = \left[x^{v\mathrm{T}}(i,j) \quad x^{v\mathrm{T}}(i,j-\tau_{v}(j)) \quad x^{v\mathrm{T}}(i,j-\tau_{vm}) \quad x^{v\mathrm{T}}(i,j-\tau_{vM}) \quad \sum_{t=1}^{d_{v}(j)} x^{v\mathrm{T}}(i,j-t)\right]^{\mathrm{T}},$$

$$\Psi_{v} = \begin{bmatrix} \Psi_{v11} & 0 & \beta^{1+\tau_{vm}}X_{v} & \beta^{1+\tau_{vM}}Y_{v} & 0 \\ & -\beta^{1+\tau_{vM}}R_{v} - 2\beta^{1+\tau_{vM}}S_{v} & \beta^{1+\tau_{vM}}S_{v} & \beta^{1+\tau_{vM}}S_{v} & 0 \\ & * & -\beta^{1+\tau_{vm}}(Q_{v}+X_{v}) - \beta^{1+\tau_{vM}}S_{v} & 0 & 0 \\ & * & * & -\beta^{1+\tau_{vM}}(X_{v}+S_{v}) & 0 \\ & * & * & * & -Z_{v} \end{bmatrix},$$

$$\Psi_{v11} = \beta Q_{v} + \beta(1+r_{\tau v})R_{v} + \beta r_{dv}Z_{v} - \beta P_{v} - \beta^{1+\tau_{vm}}X_{v} - \beta^{1+\tau_{vM}}Y_{v}.$$

From (18) and (20) we finally obtain

$$\Delta V_{\beta}(x(i,j)) = \sum_{n=1}^{8}(\Delta V_{n\beta}^{h}(x^{h}(i,j)) + \Delta V_{n\beta}^{v}(x^{v}(i,j))) \le \eta^{\mathrm{T}}(i,j)\left[\Psi + A^{\mathrm{T}}PA + D^{\mathrm{T}}\left(X+Y+S\right)D\right]\eta(i,j) \quad (21)$$

By Schur complement lemma $\Psi + A^{\mathrm{T}}PA + D^{\mathrm{T}}\left(X+Y+S\right)D < 0$

if and only if $$\Phi = \begin{bmatrix} \Psi & A^{T}P & D^{T}\Upsilon \\ & -P & 0 \\ & * & -\Pi \end{bmatrix} < 0.$$

Hence, if (9) holds then, by (21), we obtain

$$V_h(i+1, j)+V_v(i, j+1) \le \beta(V_h(i, j)+V_v(i, j)) \quad i, j \in Z^+ \tag{22}$$

For any integer $\Gamma > \kappa_0 = \max(z_1, z_2)$, one has that $V_h(0, \Gamma+1) = V_v(\Gamma+1, 0) = 0$. Then summing up both sides of (22) from $\Gamma+1$ to 0 with respect to j and 0 to $\Gamma+1$ with respect to i, one gets

$$\begin{aligned} &\sum_{i+j=\Gamma+1} V(i, j) = \sum_{i+j=\Gamma+1} \left(V_h(i, j)+V_v(i, j)\right) \\ &= V_h(0, \Gamma+1)+V_v(0, \Gamma+1)+V_h(1, \Gamma)+V_v(1, \Gamma)+\cdots+V_h(\Gamma+1, 0)+V_v(\Gamma+1, 0) \\ &= \{V_h(1, \Gamma)+V_v(0, \Gamma+1)\}+\{V_h(2, \Gamma-1)+V_v(1, \Gamma)\}+\cdots+\{V_h(\Gamma+1, 0)+V_v(\Gamma, 1)\} \\ &\le \beta\{V_h(0, \Gamma)+V_v(0, \Gamma)\}+\beta\{V_h(1, \Gamma-1)+V_v(1, \Gamma-1)\}+\cdots+\beta\{V_h(\Gamma, 0)+V_v(\Gamma, 0)\} \\ &\le \beta \sum_{i+j=\Gamma} V(i, j). \end{aligned} \tag{23}$$

From (23), using the above relationship iteratively, it follows that

$$\sum_{i+j=\Gamma} V(i, j) \le \beta \sum_{i+j=\Gamma-1} V(i, j) \le \beta^2 \sum_{i+j=\Gamma-2} V(i, j) \le \cdots \le \beta^{\Gamma-\kappa_0} \sum_{i+j=\kappa_0} V(i, j). \tag{24}$$

On the other hand, from (10), we can find two positive scalars ξ_1 and ξ_2, such that:

$$\xi_1 \| x(i, j) \|^2 \le V(i, j) \le \xi_2 \| x(i, j) \|_C^2 \tag{25}$$

Where $\xi_1 = \min\{\lambda_{\min}(P_h)+\lambda_{\min}(P_v)\}$

$$\begin{aligned} \xi_2 = \max\{&\lambda_{\max}(P_h)+\lambda_{\max}(P_v)+\tau_{hM}\lambda_{\max}(R_h)+\tau_{vM}\lambda_{\max}(R_v)+\tau_{hm}\lambda_{\max}(Q_h)+\tau_{vm}\lambda_{\max}(Q_v) \\ &+r_{dh}\lambda_{\max}(R_h)+r_{dv}\lambda_{\max}(R_v)+\tau_{hm}^2\lambda_{\max}(X_h)+\tau_{vm}^2\lambda_{\max}(X_v)+\tau_{hM}^2\lambda_{\max}(Y_h)+\tau_{vM}^2\lambda_{\max}(Y_v) \\ &+r_{\tau h}^2\lambda_{\max}(S_h)+r_{\tau v}^2\lambda_{\max}(S_v)+r_{dh}^2\lambda_{\max}(Z_h)+r_{dv}^2\lambda_{\max}(Z_v)\}. \end{aligned}$$

From (24) and (25) it follows that

$$\sum_{i+j=\Gamma} \| x(i, j) \|^2 \le N\beta^{(\Gamma-\kappa_0)} \sum_{i+j=\kappa_0} \| x(i, j) \|_C^2 . \tag{26}$$

where, $N = \dfrac{\xi_2}{\xi_1}$. Thus, the system with mixed (1) is exponentially stable. The proof is completed.

Corollary 1. *For given nonnegative integers τ_{hm}, τ_{hM}, τ_{vm}, and τ_{vM} if there exist symmetric positive definite matrices $P = diag(P_h, P_v)$, $Q = diag(Q_h, Q_v)$, $R = diag(R_h, R_v)$,*

$X = diag(X_h, X_v)$, $Y = diag(Y_h, Y_v)$, $S = diag(S_h, S_v)$, $Z = diag(Z_h, Z_v)$ *and* $0 < \beta < 1$ *such that the following LMI holds*

$$\Phi = \begin{bmatrix} \Psi & \mathcal{A}^T P & \mathcal{D}^T \Upsilon \\ & -P & 0 \\ & * & -\Pi \end{bmatrix} < 0 \tag{27}$$

where $\Upsilon = \begin{bmatrix} X & Y & S \end{bmatrix}$, $\Pi = diag(X, Y, S)$, and

$$\Psi = \begin{bmatrix} \Psi 11 & 0 & X & \bar{Y} \\ & -(\bar{R} + 2\bar{S}) & \bar{S} & \bar{S} \\ & * & -(Q + X + \bar{S}) & 0 \\ & * & * & -(\bar{X} + \bar{S}) \end{bmatrix} < 0,$$

$$\Psi 11 = \bar{Q} + R - P - X - \bar{Y},$$

$$X = I(\beta\tau_{hm}^2, \beta\tau_{vm}^2)X, \quad Y = I(\beta\tau_{hM}^2, \beta\tau_{vM}^2)Y, \quad S = I(\beta r_{\tau h}^2, \beta r_{\tau v}^2)S,$$

$$R = I(\beta(1 + r_{\tau h}), \beta(1 + r_{\tau v}))R, \quad P = I(\beta, \beta)P, \quad \bar{Q} = I(\beta, \beta)Q,$$

$$X = I(\beta^{1+\tau_{hm}}, \beta^{1+\tau_{vm}})X, \quad \bar{Y} = I(\beta^{1+\tau_{hM}}, \beta^{1+\tau_{vM}})Y, \quad \bar{R} = I(\beta^{1+\tau_{hM}}, \beta^{1+\tau_{vM}})R,$$

$$\bar{S} = I(\beta^{1+\tau_{hM}}, \beta^{1+\tau_{vM}})S, \quad Q = I(\beta^{1+\tau_{hm}}, \beta^{1+\tau_{vm}})Q, \quad \bar{X} = I(\beta^{1+\tau_{hM}}, \beta^{1+\tau_{vM}})X,$$

$$r_{\tau h} = \tau_{hM} - \tau_{hm}, \quad r_{\tau v} = \tau_{vM} - \tau_{vm},$$

$$\mathcal{A} = \begin{bmatrix} A & A_\tau & 0 & 0 \end{bmatrix}, \quad \mathcal{D} = \begin{bmatrix} A - I & A_\tau & 0 & 0 \end{bmatrix},$$

then system (1) with $A_d = 0$ *is exponentially stable.*

4. Conclusion

The exponential stability a class of 2D discrete-time systems with mixed delays has been studied in this paper. Sufficient conditions fordelay-dependent exponential stability of 2D system have been established in terms of a set of LMIs. Future work will be devoted to 2D continuous-time systems.

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